Tidal Disruption of Phobos as the Cause of Surface Fractures

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Phobos, the innermost satellite of Mars, displays an extensive system of grooves that are mostly symmetric about its sub-Mars point. Phobos is steadily spiraling inward due to the tides it raises\textsuperscript{1,2}, and will suffer tidal disruption\textsuperscript{3,4} before colliding with Mars in a few tens of millions of years\textsuperscript{5,6}. We calculate the surface stress field of the de-orbiting satellite and show that the first signs of tidal disruption are already present on its surface. Most of Phobos' prominent grooves have an excellent correlation with computed stress orientations. The model predicts an interior that has very low strength on the tidal evolution timescale, overlain by a ~10-100 m exterior shell that has elastic properties similar to lunar regolith\textsuperscript{7} or powdery asteroid materials\textsuperscript{8,9,10}.

Phobos' global network of ~10-100 m wide, kilometers-long linear features ('grooves') can be seen in Figure 1. Some resemble pit chains or graben while others resemble secondary crater chains. Some appear to radiate from the largest crater\textsuperscript{32} ~9 km Stickney; however, most grooves have no relationship to Stickney and did not form because of that impact event. Differing morphologies suggest that more than one mechanism is responsible for groove formation. Parallel grooves overprint all of Phobos' major craters, crossing their ejecta deposits, so groove formation has occurred over a timescale shorter than major crater formation, and takes place in regolith.

Phobos' bulk density\textsuperscript{11} $\rho_{av} = 1880 \text{ kg/m}^3$ and semi-major axis $a = 9376$ km place it significantly inside the Roche limit of Mars for a fluid body\textsuperscript{3}. The limited available spectroscopy\textsuperscript{12} reveals a generally asteroid-like surface composition with little, if any, evidence of hydration\textsuperscript{13}. The detailed landscape at the scale of groove formation is unknown; the best images from Mars orbiters\textsuperscript{14} are a few m/pixel locally. Thermal inertia\textsuperscript{15} indicates that the surface is mostly powdery to $\geq$0.1-1 m depth, and may extend much deeper. On the Moon, the powdery regolith is found to behave seismically as a uniform elastic layer\textsuperscript{7} to depths of ~100 m, perhaps analogous to what we find here for Phobos.

Many of Phobos' grooves feature pit chains. Several exotic mechanisms have been suggested for their formation, such as thin ejecta sheets from Mars\textsuperscript{18}, cohesionless regolith draining into dilated fractures\textsuperscript{19}, gas jets, or boulders bouncing downhill\textsuperscript{31}. While some pitted grooves resemble secondary ejecta, there is no suitable source crater on Mars or on Phobos. Apart from the physical problems with the idea, a relationship to major Martian craters\textsuperscript{18} would imply the grooves are billions of years old.

Shortly after the Viking spacecraft obtained the first geomorphic images of Phobos, it was proposed\textsuperscript{21} that stresses from orbital decay cause grooves. But, assuming a homogeneous Phobos, it proved impossible to account for the build-up of failure stress in the exterior regardless of the value assumed for Phobos' rigidity (see SOM §1). Also, stress from orbital decay follows a $1/a^3$ dependency, where $a$ is the semi-major axis (currently $2.77 \, R_{Mars}$, decaying at ~2 m per century). Hence, the greatest
tide would have built up most recently, whereas the geological record shows that the most prominent grooves range in ages, and are not always the youngest features. Hence, the tidal model languished. Here, we revisit the tidal origin of surface fractures with a more detailed treatment that shows the production of significant stress in a surface layer, with a very strong correlation to the geometry of grooves.

Tidal theory is built on the assumption that undeformed planets are spherical bodies and their tidal deformation is small compared to their radii. In Phobos’ case (13.0 km × 11.4 km × 9.1 km), the long and short axes deviate from a sphere by ~20%. The tide-raising potential acting on Phobos can be written as an expansion of Legendre polynomials. The lowest order term is a second order Legendre polynomial and is the tide-raising potential of a spherical body with average radius. We assume that perturbations on the tide-raising potential due to Phobos’ irregular shape are accounted for by higher order Legendre polynomial terms. For this study, the resulting tidal deformation from gravitational terms higher than second order are assumed to be small. We also assume that stresses due to the formation of Phobos’ irregular shape have relaxed away prior to any successive tidal deformation, and that the eccentricity of Phobos has remained historically small.

We treat Phobos as a two-layered body made up of a homogeneous interior and a discrete outer layer. Surface stresses are computed using a thin-shell spherical approximation (see SOM §2–4). The horizontal strain of this shell, as global shape evolves due to orbital decay, produces stresses on the surface given by

\[
\sigma_{\theta\theta} = \frac{9M\mu h_2}{8\pi \rho_{av}} \left( \frac{1}{a_i^3} - \frac{1}{a_f^3} \right) \left( 5 + 3 \cos 2\theta \right) \tag{1}
\]

\[
\sigma_{\phi\phi} = \frac{-9M\mu h_2}{8\pi \rho_{av}} \left( \frac{1}{a_f^3} - \frac{1}{a_i^3} \right) \left( 1 - 9 \cos 2\theta \right) \tag{2}
\]

where \( \theta \) is the colatitude measured with respect to the axis through the center of the tidal bulge \( \theta = 0^\circ \) corresponds to the sub-Mars point on Phobos’ surface. We define tension as positive and compression as negative. The surface stress \( \sigma_{\theta\theta} \) is along the surface in the \( \theta \)-direction, while \( \sigma_{\phi\phi} \) is along the surface orthogonal to \( \sigma_{\theta\theta} \).

Here \( M \) is the mass of Mars, and \( a_i \) and \( a_f \) are the starting and final semi-major axes during any span of orbital decay.

The magnitude of the tidal stress on the surface is proportional to the displacement Love number \( h_2 \), which describes Phobos’ tidal response. The value of \( h_2 \) depends on internal structure and material properties of our incompressible two-layer model (see SOM §2&4). Both layers have \( \rho_{av} = 1880 \text{ kg/m}^3 \). The inner layer has a rigidity \( \mu = 10^3 \text{ Pa} \), which approximates a strengthless material, and a total radius of 11.1 km. The outer, stiffer layer is modeled as 100 m thick based on the thickness implied by the drainage of regolith through the grooves, possibly into pore space in the
interior. This layer is modeled with \( \mu = 10^8 \text{ Pa} \), based on the assumed regolith density and knowledge of s-wave velocities that are found to be uniformly \(~200 \text{ m/s} (\mu = \rho v^2) \) in Lunar regolith\(^7\). For these model parameters \( h_2 = 0.062 \), which corresponds to a tidal bulge height of \(~70 \text{ m} \) at the sub-Mars point.

Our hypothesis, that de-orbiting tidal stress is causing surface failure on Phobos, means that to first order the principal tensile stress should be normal to the grooves. To test this, we mapped \(~200 \) of the most prominent linear surface features. Using the latitude, longitude, and strike for multiple points along these fractures, we calculated the principal tidal stress tensor experienced along each fracture in response to orbital decay, including the normal and shear stresses across each fracture. The magnitudes of the computed stresses, as well as the correlation between principal stress orientations and the azimuths of observed fractures, provide the critical tests of the tidal fracturing model.

Figure 2 shows the stress field predicted on Phobos from orbital migration starting at \( 3 R_{Mars} \) to the current orbital radius (\( 2.77 R_{Mars} \)). Tensile tidal stresses range from \( >150 \text{ kPa} \) near the sub-/anti-Mars regions to \( >30 \text{ kPa} \) at points in between. In the sub-/anti-Mars regions, both principal stresses are tensile whereas in the inter-bulge regions, stresses radial to the tidal axis are tensile and stresses concentric about the tidal axis are compressive. There is a strong correlation between the surface stress field and the geometry of Phobos’ grooves. The majority of grooves (non-purple) experience a tensile stress normal to the strike of the fault, indicating that they could have formed (or are still forming) by tensile failure of the surface. Note that we do not model failure explicitly, nor the stress perturbation associated with the opening of a fracture.

The orientations of most grooves on Phobos are closely aligned with the modeled tidal stress field. In regions of anisotropic horizontal stress, we expect the surface to fail perpendicular to the direction of maximum tension. Indeed, many of the grooves experiencing tension in these regions are aligned almost perfectly perpendicular to the direction of maximum tidal tensile stress (redder colors in Figure 2). In the sub-/anti-Mars equatorial regions, both principal stresses are tensile; however, failure will still occur perpendicular to the maximum tension as soon as the tensile strength of the surface layer is reached. Where the principal stresses have nearly equal magnitudes (i.e., approaching an isotropic stress field), no preferred formation orientation is predicted. Preferred fracture orientations in these regions are therefore less reliably compared to the tidal stress field (dark gray lines in Figure 2) and may be influenced by local sources of stress. For example, stresses from the impact that produced Stickney could have combined with tidal stresses to preferentially produce east-west grooves in the sub-Mars region near Stickney crater; or, the increased curvature around Stickney could influence the surface stress concentration.

There is a relative paucity of tidally-aligned grooves S and SE of the sub-Mars point. This might be attributed to the absence of a cohesive material in this location (e.g., coarser regolith\(^10\)). Limited spectroscopy data show regional compositional differences\(^11,13\) that might correspond to varying regolith elastic properties. Non-
aligned grooves are not random (plotted purple in Figure 2) but cluster in the equatorial zones between the sub- and anti-Mars tidal bulges; they require an alternative or additional explanation for their formation. Considering the predominant formation orientations are at high angles to the maximum compression direction, such grooves may represent contractional failure of weak surface materials, something that may require higher definition images to ascertain. Alternatively, they may have formed earlier, when Phobos was in a different tidally locked orientation. They are found predominately in the orbitally leading hemisphere, which might also allow for their formation by sweep-up of co-orbiting material derived from Phobos, Deimos, or even Mars.

Orbital decay from 3.0 to 2.77 $R_{Mars}$ can produce >150 kPa of tensile stress at the sub-/anti-Mars point (Figure 3; the 0° longitude and 0° latitude point in Figure 2). While these stresses may seem low, experiments with material designed to simulate Lunar regolith suggest that tensile failure occurs at stresses as low as 1 kPa. We find that stresses from orbital decay are likely 10–100 times larger, making deep groove formation plausible by our proposed mechanism and high enough drive fractures to depths of 1-10 km.

These stress levels could have been achieved far back in the history of Phobos' ongoing orbital decay (Figure 3), so in principle there could be generations of fractures. Grooves do not all have to be young according to our model, consistent with the apparent protracted history of their formation. As successive generations of grooves form, their orientations remain relatively consistent over time as long as Phobos' tidal alignment with Mars remains constant. Impact events might break the tidal alignment occasionally, but Phobos' irregular shape presumably helps to quickly reestablish a tidally spin-locked system.

The presence of tidally-driven fracturing on Phobos does not imply its imminent catastrophic disruption. On the contrary, a friction angle of only ~3° is sufficient to prevent downslope movement even in the complete absence of strength. But we do find that Phobos is weak enough internally, on the timescale of orbital migration, to permit tidal deformation that causes stresses to build up in an elastic lunar-like outer layer, whose failure opens up granular fissures. This interpretation is consistent with the hypothesis that the pitted grooves are formed by regolith draining into fractures.

Our model results applied to surface observations imply that Phobos is a rubble pile overlain by a lunar-like cohesive regolith layer. This layer is developing fissures as the global body deforms due to increasing tides related to orbital decay. More detailed study by an orbiter or lander will provide better constraints on the satellite's past and near-term geologic evolution, and its suitability for human exploration given what may be an active and evolving surface.


Figure 1. Phobos’ surface is covered with parallel linear features and pit/crater chains, collectively called grooves. Stickney crater is shown in the lower left of the image. Phobos has a triaxial shape of 13.0 km x 11.4 km x 9.1 km.
Figure 2. Stresses in a thin elastic shell computed for the last 10% of orbital decay, for a spherical Phobos with a nearly strengthless interior. The sub-/anti-Mars regions experience tension in both principal stress directions. Over the rest of Phobos' surface, stresses radial to the tidal axis are tensile while stresses concentric about the tidal axis are compressional. A majority of the observed fractures experience tensile stress normal to their strike (non-purple colors) while a cluster of fractures in the leading hemisphere are oriented such that the normal stress would be compressional across them (purple). The orientation of each fracture can be compared to the tidal principal stress field to assess goodness of fit. Most fractures in tension and in regions with clear large asymmetry in principal stresses are aligned nearly perfectly with the tidal stress field (warmer colors). The orientation fits of fractures in the sub-/anti-Mars regions (gray) are difficult to assess since the tidal stress is almost isotropically tensile. Any preferred formation orientations in these areas may be influenced by local stress anisotropy (Sickney crater is indicated by a dotted outline of its crater).
Figure 3. The maximum tensile stress experienced on Phobos for various amounts of orbital decay, from an orbital distance $a_i$ to $a_f$. Earlier in Phobos’ orbital history, significant stresses could be generated, allowing for several generations of fractures to evolve on its surface. The black dot represents the case illustrated in the text of migration that starts ($a_i$) at an orbital radius of $3.0R_{Mars}$ to the current orbital radius ($a_f = 2.77 \ R_{Mars}$) and results in a maximum tensile stress of slightly more than 150 kPa.
The following material serves as a reference for the reader. In general, it provides more detailed information that is supplemental to the main text. In particular the sections detail the following:

§1 Reviews the problem of assuming Phobos is a homogeneous sphere when determining tidal stresses on its surface. This finding has not been well preserved in the literature.

§2 Details how the tidal deformation of a two-layer model is determined. The standard theory here can provide the Love number $h_2$.

§3 Details the extension of the two-layer model in §2 to determine the stresses on the surface of the sphere. This method is not well documented in the literature.

§4 Discusses the evaluation of stress on the surface given by the two-layer model. It also shows how robust and sensitive the results are to model parameters. Finally, it demonstrates that a thin-shell model can approximate stresses on the surface for our nominal model parameters. Because the thin-shell equations are straightforward, we adopt them for the main text but this material serves as justification for that simplification.

1. Tidal stress on a homogeneous sphere

Surface stresses on a homogeneous sphere due to orbital migration are given by

$$\sigma_{\theta\theta} = \frac{9M\mu h_2}{20\pi \rho_{av}} \left( \frac{1}{a_f^3} - \frac{1}{a_i^3} \right)$$  \hspace{1cm} (SOM.1.1)

$$\sigma_{\phi\phi} = -\frac{9M\mu h_2}{40\pi \rho_{av}} \left( \frac{1}{a_f^3} - \frac{1}{a_i^3} \right) \left( 1 - 3 \cos 2\theta \right)$$  \hspace{1cm} (SOM.1.2)

where $\theta$ is the colatitude measured with respect to the axis through the center of the tidal bulge. Tension is positive and compression negative. The surface stress $\sigma_{\theta\theta}$ is along the surface in the $\theta$-direction, while $\sigma_{\phi\phi}$ is along the surface orthogonal to $\sigma_{\theta\theta}$.

Surface radial and shear stresses are required to be zero. Here, $M$ is the mass of Mars, $\mu$ is the rigidity, and $a_i$ and $a_f$ are the initial and final semimajor axes during orbital decay. The average density of Phobos is given by $\rho_{av}$ while its second-order tidal surface displacement Love number $h_2$ is given by the classical solution:

$$h_2 = \frac{5/2}{2\rho_{av} g R} + 1$$  \hspace{1cm} (SOM.1.3)

where $g$ is the surface gravity of Phobos and $R$ is the average radius.

In the homogeneous solution, surface stress $\sigma_{\theta\theta}$ is constant and always tensile, while
surface stress $\sigma_{\phi\phi}$ is tensile in the sub-/anti-Mars regions, extending to angular distances of 35.3°, and compressive elsewhere. Tidal stresses are maximized as $a_t \rightarrow \sim \frac{11}{5} a_f$, representing the tidal deformation from a sphere to the current tidal shape. In the case that Phobos is dominated by its material strength (i.e. $\frac{19\mu}{2\rho_{av}gR} \gg 1$), the maximum tensile stresses go to a limiting value of $\sim \frac{7GM\rho_{av}R^2}{50a^3}$, which for the parameters adopted in this study would yield about 1700 Pa (1.7 kPa). These stresses, though of the correct orientation for grooves, are not likely of a high enough magnitude for groove formation. Decreasing the rigidity of the homogenous Phobos allows greater tidal deformation but will not result in the production of greater stresses (Fig. SOM-1).

**Figure SOM-1.** Maximum tensile stress on the surface of a homogenous Phobos depends on the assumed value of its strength. For a rigid Phobos, the stress increases to a limit of $\frac{7GM\rho_{av}R^2}{50a^3}$, which for the parameters adopted in this study would yield about 1700 Pa.
2. Two-layer model of tidal deformation

In order to find a solution for the tidal deformation of a spherical two-layer body, we follow the derivation of Sabadini and Vermeersen (2004). In this method, six coupled differential equations are solved simultaneously to find the equilibrium tidal deformation. The six differential equations solve for two displacements, \( u_r \) and \( u_\theta \), two stresses, \( \sigma_{rr} \) and \( \sigma_{r\theta} \), the tide-raising potential, \( V \), and the continuity of the tidal potential, \( q \) (sometimes called the potential stress), at the top of each layer. These six equations can be written, to second order, in the following forms as:

\[
\begin{align*}
  u_r &= \eta y_1 R^2 P_2(\cos \theta), \\
  u_\theta &= \eta y_2 R^2 S_2(\cos \theta), \\
  \sigma_{rr} &= \eta y_3 R^2 P_2(\cos \theta), \\
  \sigma_{r\theta} &= \eta y_4 R^2 S_2(\cos \theta), \\
  V &= \eta y_5 R^2 P_2(\cos \theta), \text{ and} \\
  q &= \eta y_6 R^2 P_2(\cos \theta).
\end{align*}
\]

\( (SOM.2.1) \) \hspace{1cm} \( (SOM.2.2) \) \hspace{1cm} \( (SOM.2.3) \) \hspace{1cm} \( (SOM.2.4) \) \hspace{1cm} \( (SOM.2.5) \) \hspace{1cm} \( (SOM.2.6) \)

In these equations \( \eta = \frac{GM}{a^3} \) with \( a \) the orbital radius and \( G \) the gravitational constant. \( R \) is the average radius of Phobos and \( S_2(\cos \theta) \) is the derivative of \( P_2(\cos \theta) \) with respect to \( \theta \). The 6 functions \( (y_1, y_2, y_3, y_4, y_5 \text{ and } y_6) \) shown here all have the same form in that each of the terms is linear in the constants of integration such that

\[
\vec{y}_i = Y(\mu, \rho, g, r)\hat{A}
\]

\( (SOM.2.7) \)

where \( Y(\mu, \rho, g, r) \) is the matrix of functions that describe the \( y_i \) equations in terms of the constants of integration, \( \hat{A} \). As noted in Sabadini and Vermeersen (2004), the \( Y(\mu, \rho, g, r) \) matrix for spherical harmonic degree-2 is given by

\[
Y(\mu, \rho, g, r) = \begin{bmatrix}
\frac{r^3}{7} & r & 0 & \frac{1}{2r^2} & \frac{1}{r^4} & 0 \\
\frac{5r^3}{42} & \frac{r}{2} & 0 & 0 & -\frac{1}{3r^4} & 0 \\
\frac{1}{7}r^2(-\mu + \rho gr) & 2\mu + \rho gr & -\rho r^2 & \frac{-6\mu + \rho gr}{2r^3} & \frac{-8\mu + \rho gr}{r^5} & -\frac{\rho}{r^3} \\
\frac{8\mu r^2}{21} & \mu & 0 & \frac{\mu}{2r^3} & \frac{8\mu}{3r^5} & 0 \\
0 & 0 & -r^2 & 0 & 0 & -\frac{1}{r^3} \\
\frac{3}{7} \gamma \rho r^3 & 3\gamma \rho r & -5r & \frac{3\gamma \rho}{2r^2} & \frac{3\gamma \rho}{r^4} & 0
\end{bmatrix}
\]
where \( \gamma = \frac{4}{3} \pi G \).

The value of \( \vec{y}_i \) at any radius can easily be found, if the constants of integration are known, by evaluating \( Y \) at that radius. In this case, we do not know \( \vec{A} \). So, we next describe an alternative way of finding \( \vec{y}_i \) based on knowing their values at a different radius. The propagator matrix \( P \) is defined such that it gives \( \vec{y}_i \) at \( r = R_2 \) based on values at \( r = R_1 \):

\[
\vec{y}_i(r_2) = P(\mu, \rho, g_1, g_2, R_1, R_2) \vec{y}_i(r_1)
\]  
(SOM.2.9)

\( P \) is known as the “propagator” matrix. This matrix translates the solution radially within a layer in which the material properties are constant. This will allow the propagation of boundary conditions from one layer boundary to the next. The propagator matrix can be expressed in terms of \( Y(\mu, \rho, g, r) \):

\[
P(\mu, \rho, g_1, g_2, R_1, R_2) = Y(\mu, \rho, g_2, R_2) Y^{-1}(\mu, \rho, g_1, R_1)
\]  
(SOM.2.10)

At \( r = 0 \), the solution must be finite, yet there are three irregular solutions which approach infinity at this limit, allowing us to reduce the solution to three constants of integration just as Love did for a uniform sphere\(^30\). Therefore, the values of \( \vec{y}_i \) in the inner layer depend on only 3 constants of integration, and the matrix equation is reduced to

\[
\vec{y}_i(\mu_1, \rho_1, g_1, r) = Y(\mu_1, \rho_1, g_1, r) I_1 \vec{A}_1
\]  
(SOM.2.11)

for \( r < R_1 \) where \( \vec{A}_1 \) is a vector of the constants of integration for the inner layer, and

\[
I_1 = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  
(SOM.2.12)

In the model the continuity of the material requires that the displacement and potential (and thus the \( \vec{y}_i \)) must be continuous across the layer boundaries. This continuity means that at the inner layer/outer layer boundary

\[
\vec{y}_i(\mu_1, \rho_1, g_1, R_1) = \vec{y}_i(\mu_2, \rho_2, g_2, R_1) \]  
(SOM.2.13)

Next consider the outer layer. The solution anywhere in the layer \( (R_1 < r < R_2) \) can also be given by \( \vec{y}_i(\mu_2, \rho_2, g_2, r) = P(\mu_2, \rho_2, g_1, g_2, R_1, r) \vec{y}_i(\mu_2, \rho_2, g_2, R_1) \) if the solution at \( r = R_1 \) is known. However using the continuity of the solution across the boundary, the solution anywhere in this layer can also be described using the solution for the inner layer, \( \vec{y}_i(\mu_2, \rho_2, g_2, r) = P(\mu_2, \rho_2, g_1, g_2, R_1, r) \vec{y}_i(\mu_1, \rho_1, g_1, R_1) \) which now depends only on the 3 constants of integration from the inner layer. By
letting \( r = R_2 \), the solution at the surface is described in terms of three constants of integration. The solution from the inner layer/outer layer boundary at \( r = R_1 \) is said to have been “propagated” to the surface.

Now the boundary conditions at the surface can be applied in order to find the three constants of integration. These boundary conditions are the same as for a uniform sphere. The outer surface is stress free (\( y_3 = 0 \) and \( y_4 = 0 \)). Also, there is continuity in the potential across the outer surface (\( y_6 = 5/R_2 \)). Setting up the matrix equation to match the boundary conditions gives

\[
I_2 P(\mu_2, \rho_2, g_1, g_2, R_1, R_2)Y(\mu_1, \rho_1, g_1, R_1)I_1 \overrightarrow{A_1} = \overrightarrow{b} \tag{SOM.2.14}
\]

where \( \overrightarrow{b} \) is the vector of the boundary conditions on \( y_3, y_4, \) and \( y_6 \) respectively,

\[
\overrightarrow{b} = \begin{bmatrix} 0 \\ 0 \\ 5/R_2 \end{bmatrix} \tag{SOM.2.15}
\]

and \( I_2 \) is a matrix that selects only the \( y_3, y_4 \) and \( y_6 \) solutions,

\[
I_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{SOM.2.16}
\]

Solving Eq SOM.2.14 yields \( \overrightarrow{A_1} \), the set of three constants of integration

\[
\overrightarrow{A_1} = [I_2 P(\mu_2, \rho_2, g_1, g_2, R_1, R_2)Y(\mu_1, \rho_1, g_1, R_1)I_1]^{-1} \overrightarrow{b} \tag{SOM.2.17}
\]

Once the constants of integration are known, the \( \overrightarrow{y_i} \) at the surface can be fully described. The tidal response of the two-layer body as given by \( h_2 \) is related to \( y_1 \) as follows:

\[
h_2 = y_1 g_2 \tag{SOM.2.18}
\]

where \( g_2 \) is the acceleration of gravity at the surface (the top of the 2nd layer).
3. Tidal stress on the surface of a two-layer model

Even though a full solution has been found based on this propagation technique, sometimes it is useful to know the values of the six constants of integration at the surface of the outer layer. These six constants can be found by solving for them using the solution at the surface,

\[ \overrightarrow{A}_2 = Y^{-1}(\mu_2, \rho_2, g_2, R_2)P(\mu_2, \rho_2, g_1, g_2, R_1, R_2)Y(\mu_1, \rho_1, g_1, R_1)\overrightarrow{A}_1 \]  

(SOM.3.1)

This will be especially useful for calculating the surface stresses, which are given by the equations

\[ \sigma_{\theta\theta} = \eta R^2[y_7P_2(\cos \theta) + y_8S_2(\cos \theta)] \]  

(SOM.3.2)

\[ \sigma_{\phi\phi} = \eta R^2[y_7P_2(\cos \theta) + y_8T_2(\cos \theta)] \]  

(SOM.3.3)

where \( T_2(\cos \theta) \) is the derivative of \( P_2(\cos \theta) \) with respect to \( \theta \) multiplied by \( \cot \theta \). The new functions \( y_7 \) and \( y_8 \) are similar to the \( y_i \) functions in that they depend on the same constants of integration. These functions can be written in terms similar to the other functions and are

\[ Y'(\mu, \rho, g, r) = \begin{bmatrix} \frac{1}{r^2}r^2(-5\mu + \rho gr) & 2\mu + \rho gr & -\rho r^2 & \frac{\rho g}{2r^2} & \frac{2\mu + \rho gr}{r^5} & -\frac{\rho}{r^3} \\ \frac{5\mu r^2}{21} & \mu & 0 & 0 & -\frac{2\mu}{3r^5} & 0 \end{bmatrix}. \]

Thus, the \( y'_i \) functions can be evaluated by

\[ y'_i(\mu_2, \rho_2, g_2, R_2) = Y'(\mu_2, \rho_2, g_2, R_2)\overrightarrow{A}_2. \]  

(SOM.3.5)

The stress from orbital migration can be found as \( \sigma_{\theta\theta} \) and \( \sigma_{\phi\phi} \) change due to the change in orbital distance, i.e. evaluating \( \eta \) with a specific value of \( a \),

\[ \Delta \sigma_{\theta\theta} = \sigma_{\theta\theta}(a_f) - \sigma_{\theta\theta}(a_i) \]  

(SOM.3.6)

\[ \Delta \sigma_{\phi\phi} = \sigma_{\phi\phi}(a_f) - \sigma_{\phi\phi}(a_i). \]  

(SOM.3.7)

The tidal tensile stresses are maximized under the conditions that \( a_i \to \infty \), representing the tidal deformation from a sphere to the current tidal shape at \( \theta = 0 \), the sub/anti-Mars point.
4. Evaluation of a two-layer Phobos model

We use the incompressible two-layer model (SOM Section 2) to evaluate the tidal response for Phobos. The model is built of an inner layer with material properties of density $\rho_1$ and rigidity $\mu_1$ with a radial extent of $R_1$. The outer layer has its own material properties of density $\rho_2$ and rigidity $\mu_2$, but its outer radius is constrained to the average radius of Phobos, $R_2=11,200\text{m}$. The model can have up to 5 free parameters. However, to simplify things we assume that the density within the two layers is constant and set it to Phobos’ average density, i.e. $\rho_1=\rho_2=1880\ \text{kg/m}^3$. The model has now been reduced to 3 free parameters, and we further assume the rigidity of the outer layer to be $10^8\ \text{Pa}$, i.e. $\mu_2=1\times10^8\ \text{Pa}$, based on an analog to the Lunar regolith layer. We then can see how the stresses on the surface vary based on the rigidity of the inner layer $\mu_1$ and the thickness of the outer shell, $H$ (see Table SOM-1).

We find that for interiors with rigidities $<10^3\ \text{Pa}$, the stresses on the surface are well characterized by the thin-shell approximation for thinner shell thicknesses ($H<100\text{m}$). In the case of an outer shell with a thickness of 100 m, the thin-shell approximation conservatively estimates the magnitude of stress, but would produce grooves with similar orientation as long as the stresses are high enough to fracture the shell.

In the main text, we adopt a nominal model with $\mu_1=10^3\ \text{Pa}$ and an outer shell that is 100 m in thickness. The magnitude of the tidal stress is roughly inversely proportional to the shell thickness. If the outer shell thickness is 10 m, tidal stresses as shown in Figure 3 would be increased by about a factor of 10. Thus, we find that our nominal model is a conservative estimate for computing stresses on Phobos’ surface.

Even though Phobos isn’t quite spherical (it is a triaxial ellipsoid with radii of 13.0 km x 11.4 km x 9.1 km), we find that the stresses produced by a spherical shell model have orientations that match the orientations of grooves on the surface (Figure 2 in main text). Therefore, to first order, we maintain that this model can characterize stresses on Phobos due to orbital decay.

Table SOM-1: Table of two-layer model results compared to a thin-shell approximation based on the two-layer tidal deformation, $h_2$. The model evaluated consists of two layers, each with a density of 1880 kg/m$^3$. The rigidity of the inner layer is given in the table, while the outer layer has a rigidity of $10^8\ \text{Pa}$. The thickness of the outer shell ($H$) is provided in the table and the total radius is 11.2 km. The nominal model used in the text is highlighted. Table results are for the case of that starts ($a_i$) at an orbital radius of $\sim6.0R_{\text{Mars}}$ to the current orbital radius ($a_f=2.77\ R_{\text{Mars}}$).
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<th>Shell thickness $H$ [m]</th>
<th>Love number $h_2$</th>
<th>Max Tension Two-Layer Model [Pa]</th>
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Figure SOM-2. The maximum tensile stress on the surface of Phobos for various interior rigidities are shown for 2-layer models with three different outer shell thicknesses (10 m, 50 m and 100 m). The dashed lines show the thin-shell approximation for these stresses and are in good agreement with the two-layer result (solid line) as long as the interior's rigidity is $\leq 10^5$ Pa.