Meteorite transport—Revisited

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Abstract—Meteorites are delivered from the asteroid belt by way of chaotic zones (Wisdom 1985a). The dominant sources are believed to be the chaotic zones associated with the $v_6$ secular resonance, the 3:1 mean motion resonance, and the 5:2 mean motion resonance. Though the meteorite transport process has been previously studied, those studies have limitations. Here I reassess the meteorite transport process with fewer limitations. Prior studies have not been able to reproduce the afternoon excess (the fact that approximately twice as many meteorites fall in the afternoon as in the morning) and suggested that the afternoon excess is an observational artifact; here it is shown that the afternoon excess is in fact consistent with the transport of meteorites by way of chaotic zones in the asteroid belt. By studying models with and without the inner planets it is found that the inner planets significantly speed up the transport of meteorites.

INTRODUCTION

Wisdom (1985a) first suggested that meteorites may be transported directly to the Earth from the asteroid belt by way of chaotic zones, and demonstrated this though numerical simulation of the evolution of test particles near the 3:1 resonance with Jupiter. The model included perturbations from the outer planets, and used a conventional integration algorithm (Bulirsch–Stoer). Earlier hints that meteorites could be directly transported to Earth were found in Wisdom (1982, 1983), but the use of a truncated disturbing function in those works necessitated the full conventional integrations of Wisdom (1985a) to settle the matter.

The suggestion that there was an as yet unknown dynamical source of meteorites in the asteroid belt was made by Wetherill (1968). Wetherill focused on the afternoon excess exhibited by stony meteorites. The afternoon excess statistic is the ratio of the number of meteorites that fall in the afternoon (between noon and 6 P.M.) to the total daytime number (between 6 A.M. and 6 P.M.). Wetherill estimated this statistic to be $0.66 \pm 0.04$ for stony meteorites: thus about twice as many meteorites fall in the afternoon as in the morning. The falls have primarily been observed by farmers who work during the day. Furthermore, even if meteorites are seen falling at night it is harder to find them. So the statistic is restricted to daylight hours. The afternoon excess is often interpreted as saying that the meteorites are in a larger orbit and are therefore catching up to the Earth at their perihelia. But Wetherill argued that there is more information there than just that. Wetherill examined a variety of possible sources using a Monte Carlo orbital evolution scheme pioneered by Arnold (1965). He argued against the sources known at the time (lunar, Earth crossing asteroids, Mars crossers) and stated: “Thus, it appears that an Earth-crossing source for the ordinary chondrites requires the existence of unobserved bodies of low inclination with perihelia very near 1 A.U., and aphelia in the vicinity of Jupiter.” “One can also reconcile the data if there exist sources not necessarily in Earth-crossing orbits, the fragments of which nevertheless may be placed in an Earth-crossing orbit on a time scale of $10^9$ yr.”

In Wisdom (1982), I showed that objects placed near the 3:1 mean motion resonance with Jupiter could evolve for a long time at low eccentricity (of order 0.1 or less) and then suffer a “jump” to larger eccentricity (above 0.3). In the 3:1 resonance, the orbital period of the meteoroid is roughly one-third that of Jupiter. The
model used was three dimensional, and included secular variations of the orbits. However, the disturbing function was truncated to second-order terms in eccentricity. An eccentricity increase to 0.3 was sufficient to reach Mars, but Wetherill had already argued that perturbed Mars crossers could not be the source of stony meteorites. I found that the model typically gave much larger eccentricities, large enough to cross the orbit of the Earth. But because the disturbing function was truncated, it was not clear whether resonant trajectories could actually reach Earth or not. In Wisdom (1985a), I showed that they could, and the resulting orbits were just as Wetherill predicted.

In Wisdom (1983), I showed that the jumping trajectories were chaotic, and that the border of the chaotic zone coincided with the border of the 3:1 Kirkwood gap. Thus, the jumping resonant trajectories could explain the 3:1 Kirkwood gap because they are naturally removed by close encounters with the planets. In Wisdom (1985b), I developed an explanation of the basic eccentricity jumping phenomena by simplifying the problem to the planar elliptic restricted three-body problem. I showed that the jumping trajectories were moving in an unusually shaped chaotic zone with a pathway to large eccentricity. Furthermore, I showed that the shape of this chaotic zone could be explained in detail by a two time scale method for predicting the shapes of chaotic zones. Subsequently, Yoshikawa (1990) and Ferraz-Mello and Klaufke (1991) found a large chaotic region at large eccentricity in the elliptic restricted three-body problem. The extent of this chaotic region becomes larger when the secular perturbations of Jupiter’s orbit by Saturn are considered (Morbidelli and Moons 1993); the chaotic zone allows sun-grazing orbits. The interpretation was that there are overlapping secular resonances between the rate of precession of the body’s pericenter and the precession of the pericenters of Jupiter and Saturn. Note that the model in the original work of Wisdom (1982) was three dimensional and included secular variations of the orbits. However, Moons (1997) asserted that the overlapping secular resonances with the pericenter precession of Jupiter and Saturn discussed by Morbidelli and Moons (1993) explained the result of Farinella et al. (1994) that 3:1 resonance orbits typically collide with the Sun on a million year time scale. I show below that this assertion is incorrect.

Wetherill and Williams (1979) studied the transport of meteorites from the $v_6$ resonance. In the $v_6$ resonance, the pericenter precession rate matches the precession rate of the solar system mode that dominates the secular precession of the pericenter of Saturn. In their study they assumed the meteoroids underwent regular secular variations that increased their eccentricity so that the orbits became Mars crossing. Then scattering by Mars eventually led to Earth crossing on a 100 Myr time scale. But Scholl and Froeschle (1991), subsequent to Wisdom (1985a), found that there was a chaotic zone at the $v_6$ secular resonance that could directly pump up eccentricities to the point of Earth crossing on a time scale of a few million years, and thus also could be a delivery mechanism for meteorites.

Sidlichovsky (1987) and Yoshikawa (1991) studied the evolution near the 5:2 resonance. In the 5:2 resonance, the orbital period is roughly two-fifths that of Jupiter. Eccentricities are pumped up to Earth crossing very quickly in this resonance, and so it is a possible meteorite transport mechanism.

The result of Farinella et al. (1994) that many chaotic orbits in the 3:1 resonance develop eccentricities so large that they collide with the Sun is important for both the formation of the Kirkwood gaps and the delivery of meteorites. The model used in this study excluded Mercury. It used a conventional numerical integration algorithm (Bulirsch–Stoer or fifteenth order Radau). They conclude that it is more likely for a resonant chaotic body to hit the Sun than it is for the body to hit a planet or be ejected from the solar system. However, Farinella et al. (1994) incorrectly attributed the typical million year time scale to the overlapping secular resonances of Morbidelli and Moons (1993).

Gladman et al. (1997) studied the lifetimes of objects injected into asteroid belt resonances. Here I confirm some of their results. For instance, I find that the lifetimes of objects injected into the 3:1 and 5:2 resonances are similar to those they found (see below). The lifetimes of objects in the $v_6$ resonance are different, but this is probably a result of different initial conditions. However, I disagree with some of their conclusions. They state that the primary role of the inner planets is to slow the transport process down because the inner planets temporarily remove objects from the resonances. They argue that without the inner planets the development of sun-grazing orbits typically occurs on a million year time scale. Though interactions with the inner planets can indeed lead to escape from resonance, I show here that the dominant effect of the inner planets is to speed up the process leading to sun-grazing or ejection.

Gladman et al. (1997) used the Wisdom–Holman method (Wisdom and Holman 1991), as modified by Levison and Duncan (1994) to carry out their integrations and to handle close encounters. In this study they did not include Mercury. They did not mention the step size used in their study. This is important as the Wisdom–Holman method may be unstable if the step size is too large to resolve the
pericenter motion near the Sun (Rauch and Holman 1999; Wisdom 2015).

Morbidelli and Gladman (1998) studied the transport of meteorites from the asteroid belt. Specifically, they studied the transport of meteorites from the \( v_\text{e} \), 3:1, and 5:2 resonances. In their table 1, they show their results for the afternoon excess statistic for various resonances. They report the afternoon excess statistic for various collisional destruction time scales (3 Myr, 10 Myr, and infinite) and for various atmospheric entry velocity cutoffs (15 km s\(^{-1}\), 20 km s\(^{-1}\), and 31 km s\(^{-1}\)). But they conclude that no significant entry velocity bias affects the meteorite delivery process. As I do not take account of collisional destruction or a possible atmospheric velocity cutoff, the results that are most comparable to mine are the large 31 km s\(^{-1}\) entry velocity cutoff, the results that are due to and infinite collisional destruction lifetime (though my integrations were limited to 20 Myr). For these cases they found the afternoon excess for the \( v_\text{e} \) resonance to be 0.52. Curiously, they find that the afternoon excess for the 3:1 resonance to be smaller: 0.48. These numbers are substantially less than Wetherill’s estimate of 0.66 ± 0.04 for the actual falls. Indeed, they conclude “The fact that no reasonable choice of the parameters of our model allows us to recover the observed P.M. ratio of falls leads us to believe that the latter is erroneously high, perhaps due to the effect of social biases combined with small number statistics.” Considering the success of Wetherill’s deductions based on the afternoon excess statistic, the conclusions of Morbidelli and Gladman (1998) just do not have the “ring of truth.” And their results are not confirmed by my calculations, presented below.

The distribution of cosmic ray exposure ages was studied by Vokrouhlicky and Farinella (2000). They showed that the distribution is (mostly) consistent with the slow injection of meteoroids into resonances by the Yarkovsky effect. For a review of the Yarkovsky effect and its applications to asteroid dynamics see Bottke et al. (2006). Vokrouhlicky and Farinella (2000) assumed the delivery of meteorites was relatively fast and that most of the cosmic ray exposure occurred in the tens of millions of years drifting into the resonance. Their model assumes a collisional cascade during this drifting interval. They showed that a Yarkovsky drift injection mechanism is more consistent with the distribution of cosmic ray ages than is collisional injection. Up to this time collisional injection was the only mechanism known and was widely assumed. However, the story is not entirely finished, as the distribution of cosmic ray exposure ages exhibits peaks that are not explained by the Yarkovsky model. In Vokrouhlicky and Farinella (2000), some of the peaks had to be treated separately, and were assumed to be of collisional origin.

The plan of the rest of this paper is as follows. I first display histograms of the time of fall of various meteorite classes. I then turn to the dynamical simulations of the delivery from various sources. I first describe my methods. Then I show that the inner planets dramatically speed up the delivery of meteorites and the decay of the resonance population. Then I show the results of my simulations for the time of fall statistic for the principal resonances. Finally, I summarize my results.

### TIME OF FALL DATA

The time of fall statistic has played an important role in meteorite delivery studies. It has been used to rule out a variety of sources of meteorites, and it pointed to the existence of a new dynamical route for the delivery of meteorites in the asteroid belt (Wetherill 1968), which was later found (Wisdom 1985a). I have updated the histogram of the time of fall of various classes of stony meteorites using fall data from the Meteoritical Bulletin Database of the Meteoritical Society (http://www.lpi.usra.edu/meteor/), accessed September 8, 2016.

For each class of meteorite studied, the afternoon excess statistic was estimated. The estimate of the statistic and the estimate of the error was made using the method of bootstrapping (Efron and Tibshirani 1993). For a data set consisting of \( N \) entries, a resampled data set consists of \( N \) random samples from the data set, some of which may be repeated. The statistic is computed for each resampling, and the mean and standard deviation are computed from all the resampled data sets. Here I used 100 resamplings of the data. My estimate of the afternoon excess for the ordinary chondrites, 0.630 ± 0.019, is slightly less than the estimate of Wetherill (1968), 0.66 ± 0.04, but the results are compatible given the error estimates. The histogram of time of falls for ordinary chondrites is shown in Fig. 1. The histograms for the H, L, and LL chondrites are shown in Figs. 2–4, respectively.

### METHODS

The Wisdom–Holman method (Wisdom and Holman 1991) is widely used to study the evolution of planetary systems and test particles (asteroids, meteoroids, comets, etc.) perturbed by planetary systems. The Wisdom–Holman method does not handle close encounters, so in order to study the scattering of test particles off planets it must be supplemented by an encounter algorithm. A number of algorithms and codes
have been presented and used to study the evolution of test particles that may have close encounters with a collection of mutually interacting massive particles. The most popular include RMVS3 (Levison and Duncan 1994), SYMBA (Chambers 1999), and MERCURY (Chambers 1999). Each has its strengths and its
deficiencies. RMVS3 is exactly the Wisdom–Holman method in Jacobi coordinates, with a nonsymplectic method for handling close encounters. SYMBA and MERCURY are Wisdom–Holman methods using heliocentric coordinates. SYMBA handles close encounters by recursively subdividing the stepsize;

Fig. 1. Time of fall histogram for all ordinary chondrite falls. The number in the upper right corner is the afternoon excess statistic.

Fig. 2. Time of fall histogram for the H chondrite subset of falls.

Fig. 3. Time of fall histogram for the L chondrite subset of falls.

Fig. 4. Time of fall histogram for the LL chondrite subset of falls.
MERCURY makes a symplectic transition to another integration method (Bulirsch–Stoer) during encounters.

Here I use a new symplectic method for handling encounters that is based on the original Wisdom–Holman method (Wisdom and Holman 1991) in Jacobi coordinates. The method is fully described and tested in Wisdom (2016). If there is an encounter with a massive body, there is a smooth symplectic transition to another integrator. Here I use the Bulirsch–Stoer integrator.

The heliocentric variable methods, SYMBA and MERCURY, have an instability at small pericenters (Levison and Duncan 2000), as does the Wisdom–Holman method (Rauch and Holman 1999; Wisdom 2015). It appears that the instability of the heliocentric methods is worse than those methods that use Jacobi coordinates (Wisdom 2016). The instability of the Wisdom–Holman method can be avoided by using a small timestep. In most of the integrations reported in this paper, the timestep chosen was 0.05 days. This is small enough that a test particle in the 3:1 resonance can resolve a close encounter with the Sun within its Roche radius.

I did integrations both with perturbations from the outer planets only and with the full planetary system (Mercury to Neptune). I included relativistic precession in the motion of the planets when the full planetary system was used. Though a small effect, recall our result that the obliquity of Mars is not chaotic if relativity is neglected, but it is wildly chaotic if relativity is included (Touma and Wisdom 1993).

I have integrated the evolution of 1000 test particles near each of the three main resonances: the $v_6$, the 3:1, and the 5:2. In prior studies of meteorite transport, initial conditions were concentrated in the most unstable parts of the $v_6$ resonance. I chose to be less targeted. I chose random semimajor axes uniformly in the range 2.0–2.1 AU, with a Rayleigh distributed eccentricity with mean 0.1, and with a Rayleigh distributed sine of the inclination with mean 0.05. The reference plane was the J2000 ecliptic. The angles (longitude of ascending node, argument of pericenter, and mean anomaly) were uniformly distributed from 0 to $2\pi$. It turned out that all of the particles were active, in the sense that they all developed eccentricities that were large enough to encounter Mars (though not all particles had encounters). The initial conditions for the 3:1 resonance were chosen in the same way, except that the semimajor axes were uniformly distributed in the range 2.49–2.51 AU. For the 5:2 resonance, the semimajor axis range was 2.81–2.83 AU. For the 3:1 resonance, 875 particles were active, in that they both developed Mars crossing eccentricities and had encounters with the planets. For the 5:2 resonance 625 particles were active. In the plots of results only the active particles were included.

In earlier work on dynamical lifetimes (e.g., Gladman et al. 1997), it was found that the detailed initial conditions are not important, so the detailed initial conditions just described should not play an important role. Further, the typical lifetime of objects placed in these resonances had been found to be a few million years. Thus, in this study I chose to integrate the evolution for a maximum of 20 Myr. Especially for those integrations that included only perturbations from the outer planets (neglecting the inner planets) this turned out to be too short.

Collisions with the planets are relatively rare. Depending on the resonance, only a few percent of particles actually have a collision. Both Morbidelli and Gladman (1998) and I improve the collisional statistics by a statistical argument. I will first describe the procedure used by Morbidelli and Gladman (1998) and then describe the improvements made here.

Morbidelli and Gladman (1998) used the following procedure. Their method assumes that the meteoroid orbits are uniformly precessing ellipses and make the approximation that the orbit of the Earth is circular and uninclined. If the meteoroid orbit has the property that the perihelion of the meteoroid is less than 1 AU and the aphelion of the orbit is greater than 1 AU, then it is assumed that the orbits can be precessed so that there is a collision. Furthermore, it is assumed that the bodies can be positioned in their orbits so that a collision occurs. With a circular Earth orbit, the relative encounter velocity can be computed using the Tisserand parameter relative to Earth’s orbit:

$$T = \frac{1}{a} + 2\sqrt{a(1-e^2) \cos \iota},$$

where $a$, $e$, and $\iota$ are the meteoroid semimajor axis, eccentricity, and inclination. Units have been chosen so that $GM = 1$, where $G$ is the gravitational constant and $M$ is the mass of the Sun. The semimajor axis is measured in astronomical units. The encounter velocity during a collision is

$$v = \sqrt{3 - T}.$$

With the same units, the encounter velocity in the direction of the Earth’s motion is

$$v_y = \sqrt{a(1-e^2) \cos \iota - 1}.$$  

The distribution of the times of fall is then computed by assuming the meteoroid orbit is uncertain on the scale
of the Earth and there is a uniform stream of meteorites impacting the Earth on orbits parallel to the calculated orbit. Define $\theta$ through $\cos \theta = v_y/v$, then the fraction $f$ of meteoroids that fall in the afternoon (noon to 6 P.M.) relative to those that fall during the day (6 A.M. to 6 P.M.) is found to be

$$f = (1 + \cos \theta)/2.$$  \hspace{1cm} (4)

They take account of gravitational focusing, in an unspecified manner. The procedure makes numerous approximations. First, the orbit of the Earth is not circular, and it is not obvious that it is valid to replace the meteoroid by a parallel stream. But probably the most severe, and surely an incorrect assumption, is that of uniform precession. Many of the meteoroid orbits display temporary Kozai oscillations (Kozai 1962) where the eccentricity, inclination, and argument of perihelion undergo large correlated oscillations. This was already noted by Farinella et al. (1994). So it is not valid to assume that the argument of perihelion may be uniformly precessed to a collision.

I make fewer assumptions in calculating the time of fall statistic than did Morbidelli and Gladman (1998). I assumed that the position of the Earth in its orbit was uncorrelated to the position of the test particle in its orbit. So even though the particle and the Earth do not have a collision, they might have had a collision if the phases in the orbits had been different. I call these collisions “pseudo-collisions.” I let the dynamical evolution take care of the precession of the orbits; I do not assume uniform precession. Thus in my calculation I automatically take account of any correlations between eccentricity, inclination, and argument of perihelion. In more detail, I used the following procedure. The mean anomaly of the Earth was sampled uniformly in fixed increments of $2\pi/N$, where $N$ is an integer, here I chose $N = 100,000$. This is, roughly, the number of Earth diameters that fit the circumference of the Earth’s orbit. For each mean anomaly of the Earth, I found the position in the test particle orbit that gave a minimum distance between the Earth and the test particle. If this distance was less than $25 R_E$ ($R_E$ is the radius of the Earth), I backed off both orbits from the point of closest approach by a time interval of $25d/v$ where $d$ is the distance of closest approach and $v$ is the relative velocity. From this point I evolved the two bodies forward using the two-body problem and searched for collisions. Thus, I took account of gravitational focusing. For each pseudo-collision, I computed the local time of fall. I did not make the parallel streaming approximation. I did not take account of the obliquity of the Earth in computing the time of fall.

**EFFECT OF THE INNER PLANETS**

Farinella et al. (1994) found that most orbits fall into the Sun on a typical time scale of 1 Myr. They attributed this instability to the overlapping secular resonances of Morbidelli and Moons (1993). Morbidelli and Moons (1993) consider resonances between the rate of precession of the body’s pericenter and the precession of the pericenters of Jupiter and Saturn. However, the precession of Jupiter and Saturn is primarily the result of their mutual interactions—the inner planets do not contribute very much. So if it were the case that the dominant effect was secular resonances involving the precession of Jupiter and Saturn then the result should be the same whether or not the inner planets are included. But this is not the case. Figures 5 and 6 show the decay of the active particles with time with the inner planets and relativity and without the inner planets, respectively. The decay time scale with the inner planets is about 3.38 Myr, the median lifetime is about 2.35 Myr. The decay time scale of the particles without the inner planets is longer than 20 Myr. Thus, an effect involving only the secular resonances with the precession of Jupiter and Saturn cannot explain these results. The presence of the inner planets dramatically shortens the lifetime of 3:1 resonance particles.

Of the 875 active particles in the run with the inner planets, 576 had close encounters with the Sun, 247 became hyperbolic, and 12 actually had collisions with...
the planets. There were 40 particles remaining after 20 Myr. Of the 857 active particles in the run without the inner planets, 350 had close encounters with the Sun, none became hyperbolic, there were no collisions with the planets, and there were 507 particles remaining after 20 Myr. It is interesting that of the 40 orbits that remained after 20 Myr in the runs with the inner planets, only 10 evolved to semimajor axes less than 2 AU, which Morbidelli and Gladman (1998) call “evolved” orbits.

Hereinafter, I will include the inner planets and relativity in all calculations. Figures 7 and 8 show the decay of active particles in the $m_6$ and 5:2 resonances. Notice that the decay of particles in the $m_6$ vicinity is much slower than the literature would suggest (Gladman et al. 1997). Presumably this is because other studies picked their initial conditions for the $m_6$ resonance to be in the most unstable parts of that resonance, whereas I just distributed initial conditions over the resonance region, without prejudice.

**SIMULATED TIME OF FALL**

The simulated time of fall histograms speak for themselves. The simulated time of fall histogram for the 3:1 resonance is shown in Fig. 9. The afternoon excess statistic for the 3:1 resonance is $0.627 \pm 0.008$. This is close to the afternoon excess statistic for the ordinary chondrites as a whole, $0.630 \pm 0.019$. However, this must to some extent be a coincidence because it seems likely that different classes of meteorites have different injection histories (based on the distribution of cosmic ray exposure ages) and possibly have contributions from different resonance transport mechanisms.

The simulated time of fall histogram for the $v_6$ resonance (Fig. 10) exhibits a markedly lower afternoon...
excess statistic of 0.544 ± 0.004. By itself this would suggest that more ordinary chondrites are delivered through the 3:1 resonance than through the $m_6$ resonance. But again, there may be contributions to different classes of meteorites from different mechanisms.

The simulated time of fall histogram for the 5:2 resonance (Fig. 11) has an afternoon excess statistic of 0.679 ± 0.026. This is somewhat larger than the 3:1 statistic. In computing this histogram and this statistic I doubled the number of simulated trajectories to 2000 (from 1000) to improve statistics.

**SUMMARY**

I have updated the histograms of the time of fall of the ordinary chondrites, as well as the H, L, and LL chondrites.

I use a model that includes all the planets from Mercury to Neptune, and the relativistic precession of their orbits. Prior studies excluded Mercury and relativity. I use a stepsize that is small enough to avoid integrator instabilities.

I have shown that the inner planets significantly speed up the processes of meteorite transport and the decay of resonant populations.

I have found through simulation the expected time of fall histograms for the $v_6$, 3:1, and 5:2 resonances, and estimated the afternoon excess statistic for each.

Overall, the data from both observation and simulation are consistent with a source of meteorites in the middle of the asteroid belt, as deduced by Wetherill (1968).

It would be nice to be more specific about which types of meteorites come from which resonances, but the data at present have estimated errors that are too large to permit this. The data suggest, however, that the 3:1 resonance chaotic zone plays an important role in
transporting meteorites from the asteroid belt to the Earth (Wisdom 1985a).

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REFERENCES


