The characteristics of magma reservoir failure beneath a volcanic edifice

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Eruptions fed from subsurface reservoirs commonly construct volcanic edifices at the surface, and the growth of an edifice will in turn modify the subsurface stress state that dictates the conditions under which subsequent rupture of the inflating reservoir can occur. We re-examine this problem using axisymmetric finite element models of ellipsoidal reservoirs beneath conical edifices, explicitly incorporating factors (e.g., full gravitational loading conditions, an elastic edifice instead of a surface load, reservoir pressures sufficient to induce tensile rupture) that compromise previous solutions to illustrate why variations in rupture behavior can occur. Relative to half-space model results, the presence of an edifice generally rotates rupture toward the crest of a spherical reservoir, with increasing flank slope (for an edifice of constant volume) and larger edifices (or greater reservoir scaled depths) normally serving to enhance this trend. When non-spherical reservoirs are considered, the presence of an edifice amplifies previously identified half-space failure characteristics, shifting rupture to the crest more rapidly for prolate reservoirs while forcing rupture closer to the midpoint of oblate reservoirs. Rupture is always observed to occur in the ω orientation, and depending on where initial failure occurs rupture favors the initial emplacement of either lateral sills, circumferential intrusions or vertically ascending dikes. Ultimately, integration of our numerical model results with other information, for instance the sequence of intrusion/eruption events observed at a given volcano, can provide useful new insight into how a volcano’s subsurface magma plumbing system evolved. We demonstrate this process through application of our model to Summer Coon, a well-studied stratocone on Earth, and Ilythia Mons, a large conical shield volcano on Venus.

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1. Introduction

A volcano is a complex system that extends into the subsurface and can include a pressurized magma reservoir that ruptures to yield vertically or laterally propagating intrusions. Depending on their orientation, the initial fissures can produce surface eruptions, vertical or circumferential intrusions, subsurface sills, linear dike systems, or radiating dike swarms (e.g., Rubin and Pollard, 1987; Walker et al., 1995; Gudmundsson, 1998; Klausen, 2004). Complementing field and laboratory studies, quantitative models that simulate subsurface volcanic systems can be used to understand the conditions that favor one style of intrusion over another. Many previous models, for example, use a spherical reservoir in a half-space to study subsurface magma propagation, surface deformation, or other processes (e.g., Dieterich and Decker, 1975; Bianchi et al., 1984; Bonafede et al., 1986; McGtigue, 1987; Chevallier and Verwoerd, 1988; Gudmundsson, 1988; Tait et al., 1989; Quareni, 1990; Chadwick and Dieterich, 1995; Koenig and Pollard, 1998; Muller et al., 2001; Newman et al., 2001; Trasatti et al., 2003; Ellis et al., 2007; Manconi et al., 2007). However, most previous studies do not address how the complex interplay between reservoir inflation and host rock stress state will control rupture of the reservoir wall. Rupture location and orientation will have implications for the type of intrusion created, and this knowledge may be critical to our understanding of how volcanic edifices initiate and evolve. Several recent studies have explicitly investigated the factors controlling reservoir rupture using either analytical or numerical techniques. Most of these focus on half-space configurations that do not consider the effects of surface topography (e.g., Sartoris et al., 1990; Parfitt et al., 1995; Gudmundsson, 1998; Grosfils, 2007; Long and Grosfils, 2009) while others incorporate an edifice, either to address circumstances specific to an individual volcano or to investigate how edifice loading affects reservoir rupture conditions (e.g., Russo et al., 1997; Pinel and Jaupart, 2003; Marti and Geyer, 2009). Interpretation and application of the latter studies, however, is complicated by a variety of issues. For example, in their study of Vesuvius Russo et al. (1997) set the uniform component of the magma pressure in the reservoir to a fixed value equal to the lithostatic stress at the crest, but they do not systematically assess the conditions under which variations in this pressure (or edifice geometry) will actually lead to tensile rupture. Both Pinel and Jaupart (2003) and Marti and Geyer (2009) adopt a more parameterized approach, but their condition for rupture (dike injection) is derived from a normal stress balance across the...
reservoir wall. This approach, employed in many magma reservoir models, does not correctly incorporate the wall-parallel components of the lithostatic stress when assessing failure, compromising interpretation of the pressure required to induce tensile failure, the affiliated wall displacement, and hence the rupture location (for a detailed discussion, see Grosfils, 2007). These circumstances call into question our current understanding of how the presence of an edifice affects the rupture characteristics of an underlying, inflating reservoir.

In the current study we re-examine how the presence of an edifice will affect the rupture characteristics of an underlying magma reservoir when full gravitational loading conditions are explicitly incorporated. Regional tectonic stresses, such as those treated by Russo et al. (1997) and Russo and Giberti (2000, 2004), are omitted in order to isolate the effects an edifice alone will have on rupture behavior. We begin by investigating the effect that edifice slope and reservoir:edifice volume ratios have on the rupture behavior of a spherical reservoir, then expand upon this treatment by exploring the rupture characteristics of other ellipsoidal reservoir geometries. We compare our parameterized results directly with those of Pinel and Jaupart (2003), and use a sample edifice/reservoir configuration to illustrate how inclusion of the full gravitational stress affects interpretation of the rupture location and pressure conditions required. Finally, we illustrate how our numerical method and results can be used to gain improved insight into the volcanic evolution of two distinctly different edifices: Summer Coon, an eroded stratocone in Colorado that has already been the subject of intensive field study, and Ilithyia Mons, a large shield edifice on Venus for which only remote sensing data are available. While more sophisticated models incorporating regional stress variations and other factors are certainly a necessary future step, our results provide useful new insight into magma reservoir failure as well as an internally self-consistent foundation upon which future models can be built in an effort to improve our understanding of volcanic activity and edifice growth on Earth, Venus and other solar system bodies.

2. Methods

2.1. Model setup

Following the approach described in detail by Grosfils (2007), we use COMSOL Multiphysics (http://www.comsol.com/) to create an elastic, axisymmetric finite element geometry (Fig. 1). Magma density ($\rho_m$) and host rock density ($\rho_h$) are set equal to one another at a value of 2700 kg m$^{-3}$ [except where otherwise stated], appropriate for generally basaltic material, and we use a Poisson’s ratio of 0.25 and Young’s modulus of 2.5 GPa. The initial host rock is defined by a rectangular section, from which a semi-elliptical section (vertical radius $R_e$, horizontal radius $R_h$, center at depth $D_C$ below the z = 0 surface) is cut and upon which a triangular edifice slice (central height $H_e$, basal radius $R_b$ and slope $\gamma = H_e/R_b$) is superimposed. The integrated section is rotated about the z-axis to define the axisymmetric volume, with the axis of rotation bisecting the resulting ellipsoidal reservoir and conical edifice; the edifice shape, though idealized, nevertheless captures a geometry that is characteristic of many volcanoes (e.g., Grosse et al., 2009). The ellipsoidal reservoir has an aspect ratio $a = R_b/R_e$, where $a > 1$ represents a prolate reservoir, $a < 1$ represents a spherical reservoir, and $a < 1$ represents an oblate reservoir; when studying spherical reservoirs, i.e. when $R_b = R_e$, we simply denote the reservoir radius as $R_e$. The upper surface of the model is left free to displace, simulating a planetary surface, while the lower and outer edges of the model are respectively constrained to permit only lateral and vertical displacements (i.e., roller conditions). The mesh consists of tens of thousands of triangular, second-order Lagrangian elements that decrease in size near the reservoir wall and free surface in order to provide maximum resolution near these boundaries of interest. The smallest elements are approximately 100 m in length for a reservoir of radius $R_e = 4000$ m and 2 m in length for a reservoir of $R_e = 200$ m, resulting in a model uncertainty of $\sim 2\%$ when identifying rupture location.

The host rock, simulating material in which stresses have relaxed to equilibrium over time, is assigned a body load per unit volume of $F_z = -\rho_g g$, where $g$ is gravitational acceleration, and an initial lithostatic pre-stress of $\sigma_z = \rho_h g H_e$ to $\rho_h g H_e = 0$) and $\sigma_r = \rho_h g R_e$ (when $r > R_e$, $\sigma_r = 0$). The host rock is defined by $\sigma_z = \rho_h g H_e$ (when $z > H_e$, $\sigma_z = 0$). The host rock is defined by $\sigma_z = \rho_h g H_e$ (when $z > H_e$, $\sigma_z = 0$).

Physically, this simulates edifice formation that occurs rapidly relative to the time required for the underlying substrate material to adjust. An alternative formulation would be to examine a situation in which the edifice has been present for enough time that the entire system, including the host rock, has returned to a state of lithostatic equilibrium described by $\sigma_z = \rho_h g H_e$ (when $z > H_e$, $\sigma_z = 0$) for the host rock, but this approach also incorporates limitations (e.g., stress reflects a mature system that has already adjusted to the edifice load emplacement, but the model does not initially incorporate any of the associated displacements that would have occurred). The most ideal model configuration likely lies somewhere between these two endmembers. Careful comparison between them under a wide array of model conditions indicates a difference in rupture predictions of roughly 2° or less in nearly every instance, effectively within the uncertainty of the model, and the differences observed always enhance the trends we identify in Section 3. Given the minor nature of the differences in outcome, we choose to adopt the “rapid edifice emplacement” model simply because it permits the best direct comparison with most previously published results (e.g., Pinel and Jaupart, 2003) in our models we ignore regional tectonic stresses (such as those explored by others; e.g., Russo et al., 1997, Marti and Geyer, 2009) in order to isolate and better understand the effects an edifice load will have on rupture behavior.

For the reservoir, boundary conditions imposed normal to the walls simulate internal pressurization of the chamber, and following previous studies (e.g., Parfitt et al., 1993; Russo et al., 1997; Grosfils, 2007) these are given by:

$$\alpha_h = -P + \rho_m g H_m.$$  

(1)
The term $P$ represents a uniform, depth-independent pressure that incorporates the minimum pressure required to keep the reservoir open and any additional depth-invariant factors, such as a pressure increase caused by an influx of magma from the mantle or the fractionation of volatiles within the reservoir. The term $\rho_0 g h_m$ quantifies the depth-dependent pressure from the magma weight with $h_m$, the distance below the crest of the magma reservoir, negative downwards like $z$ in the host rock (see Fig. 1). The combination of these two pressure terms creates an internal reservoir pressure applied normal to the reservoir wall that increases linearly with depth, ranging in magnitude from $P$ at the crest of the reservoir ($h_m = 0$) to $P + 2\rho_0 g R_e$ at the base ($h_m = -2R_e$).

2.2. Model parameters

In general we employ non-dimensional parameters to define the geometric and physical characteristics of the numerical model in order to produce more broadly applicable results. Specifically, we characterize the model using the parameters $R_e/D_C, H_e/D_C, R_o/D_C$, and $R_o/R_e (a)$, choosing $D_C$ as the primary length scaling parameter to promote comparison with previously published half-space models. A dimensionless edifice slope $\gamma (H_e/R_e)$, the ratio between $H_e/D_C$ and $R_o/D_C$, is also used to describe the non-dimensional edifice shape concisely. The ratio $R_o/D_C$ is used as a dimensionless reservoir depth, where $R_o/D_C=0.1$ represents a ‘deep-seated’ reservoir and $R_o/D_C=0.9$ represents a ‘shallow’ reservoir. By varying four parameters — $\gamma$ to determine the effect of edifice slope, $R_e$ and $R_o$ to determine the effect of a spherical magma chamber volume ($V_e$) relative to the edifice volume ($V_e$), and $a$ to control the reservoir geometry — we can explore the interplay between edifice- and reservoir-derived stresses and the corresponding effects on reservoir rupture behavior.

2.3. Rupture of the reservoir

To simulate failure of an internally pressurized reservoir, the magnitude of the uniform pressure $P$ is increased until rupture of the wall first occurs at some angle $\theta$, where $\theta = 0^\circ$ corresponds to the reservoir crest and $\theta = 90^\circ$ to the midpoint. Rupture of the reservoir wall occurs when either the horizontal hoop stress ($\sigma_h$) or the wall-parallel stress within a vertical plane ($\sigma_z$) first exceeds the tensile strength limit (TSL) of the surrounding host rock. Reflecting weak rock mass strength (i.e., Schultz, 1995) we assume TSL $<$ 1 MPa for convenience unless otherwise noted; tests reported in Grosfils (2007) show that selection of any TSL in the 0–10 MPa range generally has only a minimal effect on the failure behavior. The two possible stress orientations, shown in Fig. 1, yield initial fractures (and hence intrusions) of different orientations: failure in the $\sigma_h$ orientation will create fractures favoring initial emplacement of vertical dikes ($\theta = 0^\circ$), circumferential intrusions, or sills ($\theta = 90^\circ$), whereas failure in the $\sigma_z$ orientation will initially yield either vertical ($\theta = 0^\circ$) or radially-configured lateral dikes. The depth-invariant pressure required to induce rupture ($P_{\text{fail}}$) is also compared to the initial lithostatic pressure ($P_{\text{li}} = \rho g z$) at the reservoir $D_C$ to ensure that results are geologically plausible. An initial reservoir geometry is considered geologically implausible if the magnitude of $P_{\text{fail}}$ is less than the minimum pressure required to keep the reservoir open against the surrounding lithostatic stress at the presumed depth of reservoir nucleation/growth (i.e., $D_C$).

The reservoir wall is assumed to have no unusual stress-concentrating defects, leading the model to approximate a maximum rupture pressure. Local inhomogeneities are of course likely to alter the nature of the failure process by creating irregular concentrations of stress in specific cases (e.g., Letourneau et al., 2008); however, field observations (i.e., persistent dominance of circumferential and radial intrusions fed from shallow magma sources) show a broad concordance between reservoirs and their associated intrusion geometries across a wide range of geological settings, suggesting that patterns of magma reservoir failure and intrusion geometry are not dictated primarily by such inhomogeneities. A more idealized analysis of the wall-parallel stress magnitudes (cf. Russo et al., 1997) is thus expected to provide useful insight into the anticipated nature of reservoir rupture and any corresponding magmatic and/or volcanic activity.

3. Results

A general analysis of magma reservoir rupture behavior, explored first for edifices of different sizes and geometries above spherical reservoirs and then for reservoirs of varied geometries beneath an edifice of constant dimensions, reveals that rupture consistently occurs either at the crest (where $\sigma_h$ and $\sigma_z$ have equivalent magnitudes) or, at all other locations, in the $\sigma_h$ orientation. Failure near the crest thus promotes vertical propagation of radially aligned dikes, failure near the mid-depth of the reservoir produces lateral sill injection, and rupture in between favors formation of circumferential intrusions. In addition, the location of maximum tensile stress corresponds closely to the location of maximum strain deviation $e_p$, defined by Jaeger and Cook (1979) as the difference between the strain $e_q$ in the $\sigma_h$ direction and $e_r$, the mean normal strain ($e_t = e_r - e_q$). These results are consistent with those reported by Grosfils (2007).

3.1. Edifice effects on rupture of spherical reservoirs

To isolate how loading from an edifice affects the rupture behavior of an underlying spherical reservoir, we investigate two sets of conditions. First, the edifice slope is varied ($\gamma = 0.01$ to 0.23) while the reservoir geometry ($R_e/D_C$) and edifice volume are kept constant. In these model runs the reservoir:edifice volume ratios are also held constant ($V_e/V_e \approx 30$, simulating initial edifice growth above a mature, established reservoir), and hence the parameters $R_e/D_C, H_e/D_C$, and $R_o/D_C$ are also invariant for a given $\gamma$. This tests how altering the distribution of the surface load imposed by an edifice modifies failure at the underlying reservoir wall. Second, in order to examine how the relative volumes of the reservoir and edifice interact to modify an inflating reservoir’s failure characteristics, we hold the edifice geometry and volume as well as the reservoir depth constant while varying the reservoir radius. Finally, for small and large reservoirs of fixed size beneath edifices of fixed flank slope, we assess how the interplay between edifice radius and scaled reservoir depth affects rupture behavior relative to half-space conditions.

3.1.1. Edifice geometry: constant volume, variable slope

The effects of the edifice slope $\gamma$ on the rupture behavior of a spherical reservoir are shown in Fig. 2. As $\gamma \rightarrow 0$ an edifice of constant volume becomes broader and thinner, and the distribution of the mass should thus increasingly resemble a half-space model scenario. For a deep-seated reservoir ($R_e/D_C = 0.3$) rupture in a half-space model occurs at the crest of the reservoir (Grosfils, 2007). Fig. 2 shows that rupture of a deep-seated reservoir continues to occur at this location independent of how the edifice mass is distributed. The presence of the edifice is thus not expected to affect the failure characteristics of a source reservoir at great depth, and failure of such a reservoir due to inflation is expected to continue promoting vertical ascent of magma via dikes.

In contrast to the deep-seated reservoir case, the presence of an initial edifice clearly modifies the failure characteristics of reservoirs located closer to the surface. For a half-space scenario in which $R_e/D_C = 0.5$ or 0.7, failure occurs roughly 30° from the crest. When an edifice load is introduced, however, Fig. 2 illustrates that the failure location rapidly shifts toward the reservoir crest as $\gamma$ increases. If the edifice mass is distributed in a shield-like configuration, failure
continues to occur 20–30° from the crest, promoting circumferential intrusion. As \( \gamma \) becomes greater than \( \sim 0.1 \), i.e. once the edifice slopes exceed \( \sim 6° \) (a typical value for a basaltic shield volcano, cf. Pinel and Jaupart, 2003), failure becomes locked at the crest. Initial growth of a more composite cone-like edifice above a shallow reservoir, therefore, promotes continued vertical dike intrusion when reservoir rupture occurs.

3.1.2. Variation in reservoir-to-edifice volume

To explore how variations in the reservoir:edifice volume ratio affect failure we employ models in which reservoir \( D_t/C \) is held constant at a value equal to either \( R_c \) or \( 2R_c \) beneath two different edifice geometries: (a) \( R_c =3000 \text{ m}, \gamma =0.1 \) (so \( V_e =2.83 \times 10^8 \text{ m}^3 \)), and (b) \( R_c =6000 \text{ m}, \gamma =0.1 \) (\( V_e =2.26 \times 10^{10} \text{ m}^3 \)). The spherical reservoir radius \( R_c \) is then varied to generate volumes \( V_c \) that range from 0.03 to 150 times \( V_e \). In these models, the dimensionless parameters \( R_c/D_t/C \) and \( \gamma \) are held constant for a given \( R_c/D_t/C \).

Fig. 3 shows the changes in rupture behavior of reservoirs with \( R_c/D_t/C =0.5 \). Reservoirs with volumes less than \( \sim 33V_e \) rupture consistently at the reservoir crest; this occurs because the edifice is too deep to affect the rupture behavior. When \( 0<\gamma<0.1 \), the edifice effect that generally pins rupture at the crest diminishes and rupture approaches the half-space rupture behavior (shaded symbols at \( \gamma =0 \)). Rupture behavior remains consistent between models scaled by a constant factor within the uncertainty of the model (2°), in this case a factor of 20 between \( R_c =200 \text{ m} \) and \( R_c =4000 \text{ m} \).

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Fig. 2. The effect of edifice mass distribution on spherical reservoir rupture. This graph shows the rupture behavior of spherical reservoirs of \( R_c =200 \text{ m} \) and 4000 m (solid and hollow symbols, respectively) at \( R_c/D_t/C =0.3, 0.5, \text{ and } 0.7 \) (triangle, circle, and square symbols, respectively) beneath an edifice of constant volume \( V_e =1.25 \times 10^9 \text{ m}^3 \) for \( R_c =200 \text{ m} \) or \( V_e =1 \times 10^{10} \text{ m}^3 \) for \( R_c =4000 \text{ m} \). The volumes were chosen to maintain a constant (arbitrary) reservoir:edifice volume ratio of \( V_c/V_e =26.8 \). A reservoir at great depth, i.e. \( R_c/D_t/C =0.3 \), ruptures consistently at the crest when placed both beneath an edifice and within a half-space (half-space condition shown by shaded triangle at \( \gamma =0 \)), indicating that the reservoir is too deep for rupture behavior to be affected by the edifice. Reservoirs at shallower depths, i.e. \( R_c/D_t/C =0.5 \text{ and } 0.7 \), rupture at the crest when placed beneath an edifice with \( \gamma =0.1 \) rather than at 30° when placed within a half-space. When \( 0<\gamma<0.1 \), the edifice effect that generally pins rupture at the crest begins to diminish and rupture approaches the half-space rupture behavior (shaded symbols at \( \gamma =0 \)). Rupture behavior remains consistent between models scaled by a constant factor within the uncertainty of the model (2°), in this case a factor of 20 between \( R_c =200 \text{ m} \) and \( R_c =4000 \text{ m} \).

Fig. 3. The effect of edifice volume \( (V_c/V_e) \) on spherical reservoir rupture. This graph displays the resulting rupture behavior of spherical reservoirs of different radii set at depths corresponding to \( R_c/D_t/C =0.5 \) (circles) beneath a constant edifice in order to compare the rupture behavior of reservoirs at each depth (constant \( R_c/D_t/C \) yield equivalent results within the uncertainty of the model, 2°). The graph shows that reservoirs smaller than \( 33V_e \) rupture only at the crest, but once the reservoir volume exceeds \( 33V_e \) rupture shifts away from the crest. These results indicate that, as the edifice grows relative to a constant reservoir volume, the effect of the edifice to pull rupture towards the reservoir crest dominates the free surface effect to push rupture towards the reservoir midpoint as expected in the half-space case (line with diamonds). Dashed lines indicate unreasonable geometries in which the reservoir would not be expected to form because \( |P_{\text{bull}}|>|P_{\text{fail}}| \); see text for details.
rupture also occurs only at the crest for cases where \( V_r < 33 V_c \); mathematically, a spherical reservoir at this \( D_e \) will intersect the free surface if it grows much larger, and we consider anything close to this situation to be physically unlikely. As the reservoir volume increases past \( \approx 33 V_c (R_e/D_e = 0.46) \), however, rupture shifts away from the reservoir crest. With further increases in reservoir volume, as the effective size of the edifice shrinks the model approaches but does not reach the behavior expected for a spherical reservoir when an edifice is absent. In general, when compared with results from half-space models that lack an edifice, we conclude that rupture of a reservoir located beneath an edifice remains more effectively locked at or near the reservoir crest. Put another way, as the size of a shield-like edifice increases while the reservoir geometry remains constant, a tendency to favor circumferential intrusions will transition to vertical dike emplacement, promoting central eruptions and stabilizing the continued growth of the edifice.

3.1.3. Growth of shield volcanoes and stratocones

To gain insight into variations that occur during the growth of edifices with shield and stratocone morphologies, we expand our analysis by assessing failure for spherical reservoirs of either \( R_e = 200 \, \text{m} \) or \( R_e = 4000 \, \text{m} \) placed beneath edifices with \( \gamma = 0.1 \) (shield volcano) or \( \gamma = 0.6 \) (stratovolcano). These parameter selections also promote direct comparison with the widely cited analytical model results published previously by Pinel and Jaupart (2003), and here we thus adopt their choice of \( T_S = 20 \, \text{MPa} \) as well. We then compare the rupture location for a reservoir beneath an edifice (angle \( \theta_{ed} \), equivalent to Pinel and Jaupart’s \( \theta_i \)) with that of an equivalent reservoir in an edifice-free half-space (angle \( \theta_{hs} \), equivalent to Pinel and Jaupart’s \( \theta_h \)) as the dimensionless depth ratio \( R_e/D_e \) is varied. We plot the results in two ways, first to facilitate direct comparison with the graphs produced by Pinel and Jaupart (2003) and second to quantify the difference between \( \theta_{hs} \) and \( \theta_{ed} \). Ultimately, there are two key differences between our approach and that of Pinel and Jaupart (2003). The first is that we build the edifice ‘physically’ in our models using elastic elements instead of simply applying a mathematical loading function at the surface to simulate the edifice weight. This difference is subtle but potentially important because, even though the vertical load applied will be identical, the additional edifice material is itself capable of a full elastic response, and hence will modulate the host rock displacement and stress fields that would otherwise occur at the free surface. Second, our FEM formulation ensures that critical wall-parallel components of the lithostatic stress field are incorporated when calculating failure location; the significant errors that can be introduced by using the Jeffery (1921) solution, a common approach and one adopted by Pinel and Jaupart (2003), are assessed in detail by Grosfils (2007).

Fig. 4 displays our results, organized into four sets of panels. The left-most panel i in each set shows the results from Pinel and Jaupart (2003; their Figs. 12 and 13), while the center panel ii shows data from the current study in the same format to promote direct comparison. We identify rupture changes in which \( \theta_{ed} < \theta_{hs} \), \( \theta_{ed} > \theta_{hs} \), or \( \theta_{ed} = 0 \), the three outcomes predicted by Pinel and Jaupart (2003), but our results suggest that these shifts in rupture can occur under somewhat different conditions than those predicted previously. In addition, we recognize a fourth case: \( \theta_{ed} = \theta_{hs} \) with \( \theta_{ed} \neq 0 \), where the ‘equals’ condition is defined as \( \theta_{hs} \pm \theta \), reflecting a conservative view of the general precision of our models. This fourth outcome, evident prominently across the current study’s panel ii results, is an important and previously neglected relationship denoting circumstances in which the presence of an edifice has essentially no effect on a rupture location away from the crest.

The right-most panel in each set of data shown in Fig. 4, iii, displays our results in a format that quantifies the magnitude of the shift in rupture location that is caused by addition of an edifice; grey-shaded areas show where this shift (relative to half-space model results) is less than 5° (\( \left| \theta_{ed} - \theta_{hs} \right| < 5 \)), i.e. conditions under which only minor changes in rupture location occur. The numbers within each plot iii indicate the difference between \( \theta_{ed} \) and \( \theta_{hs} \); while those shown along the top edge indicate where failure would occur for the same \( R_e/D_e \) within an edifice-free half-space (\( \theta_{hs} \)). To illustrate use of these data, consider the column in Fig. 4a iii where \( R_e/D_e = 0.9 \), i.e. an extremely shallow reservoir. At this value of \( R_e/D_e \), failure in the absence of an edifice (shown at top edge of graph) occurs at \( \theta_{hs} = 26° \), i.e. 26° from the crest. As a small edifice gradually forms (\( R_e \) increases), stress changes induced at the reservoir walls promote failure closer to the edifice midpoint than in the half-space case, e.g. for the smallest edifice modeled the shift is 15°, so \( \theta_{ed} = 26 + 15 = 41° \) from the crest. As the edifice exceeds a certain threshold radius however, which occurs in this case when \( R_e \approx 2 \, \text{km} \), failure rapidly shifts to the crest of the reservoir, i.e. \( \theta_{ed} = 26 + (-26) = 0 \), where it remains as edifice construction continues. One volcanicological implication is thus that, while initial growth of the edifice has the tendency to induce rupture that generates circumferential intrusions (and presumably flank eruptions) from a small shallow reservoir, once an edifice of sufficient size forms, reservoir failure shifts abruptly to the crest, promoting vertical dike injection from this location and movement of magma toward the surface. We explore the broader patterns of rupture and the volcanicological implications of Fig. 4 further in Section 4.1.

3.2. Edifice effects on the rupture of other ellipsoidal reservoir geometries

Thus far only spherical reservoir geometries have been analyzed, but rupture of spherical reservoirs does not occur at locations or in orientations that promote initial intrusion of lateral dikes or sills, thus failing to explain two sets of common volcanic features. In an attempt to reproduce these features, previous studies have explored rupture behavior of ellipsoidal reservoirs within a half-space to a limited extent (e.g., Russo et al., 1997; Grosfils, 2007; Marti and Geyer, 2009). The current study expands upon these previous works, also to a limited extent, to investigate rupture behavior of ellipsoidal reservoirs beneath an edifice. The following two subsections describe how rupture behavior changes for prolate and oblate reservoirs with constant volumes equivalent to a spherical reservoir with \( R_e = 4000 \, \text{m} \); all are placed beneath an edifice of \( H_e = 625 \, \text{m} \) and \( R_e = 54.5 \, \text{km} \). In one sense this choice of reservoir volume and edifice size is arbitrary, intended solely to illustrate key aspects of the failure behavior, but we also select it in order to promote later application to an analysis of the magmatic evolution of Ilithya Mons on Venus (see Section 4.2.2).

3.2.1. Prolate reservoir

Two prolate reservoirs of constant volume (\( V = 268 \, \text{km}^3 \), equivalent to a sphere with \( R_e = 4000 \, \text{m} \)) and of aspect ratios \( a \) are tested beneath the edifice, with \( a = R_p/R_e \) set to either 1.1 (\( R_p = 4250 \, \text{m}, R_e = 3880 \, \text{m} \)) or 1.4 (\( R_p = 5050 \, \text{m}, R_e = 3560 \, \text{m} \)); \( R_p \) is used to determine the reservoir’s scaled depth (\( R_p/D_e \) beneath the surface. Resulting rupture locations (\( \theta_{ed} \), see equation 19 in Grosfils, 2007), plotted in Fig. 5, demonstrate that reservoirs with greater vertical elongation (i.e. greater aspect ratio) must lie at shallower depths (larger \( R_p/D_e \) ratios) to shift rupture away from the crest regardless of whether or not an edifice exists. This tendency is further enhanced when an edifice is present. For example, rupture shifts away from the crest of a spherical reservoir beneath Ilithya Mons when \( R_p/D_e = 0.47 \); however, rupture shifts away from the crest of a reservoir with an aspect ratio of 1.1 near \( R_p/D_e = 0.65 \), and rupture never shifts from the crest when the aspect ratio is 1.4. Increasingly prolate reservoir geometries, like addition of an edifice load, thus tend to help promote vertical dike injection.

As in the spherical reservoir results reported in Figs. 3 and 4, not all prolate reservoir model runs are geologically reasonable. In general, as
Fig. 5. Rupture behavior for prolate reservoirs. These reservoirs have the same volume as a sphere with \( R_e = 4000 \) m and have a vertical semi-major axis of radius \( R_a \), a horizontal semi-minor axis of radius \( R_b \), and aspect ratio \( a = R_a/R_b \). A conical volcano \( (H_v = 625 \) m, \( R_e = 543.5 \) km). Rupture angles are plotted for two prolate reservoir (\( a = 1.4 \), triangle line; \( a = 1.1 \), diamond line) and a spherical reservoir (\( a = 1.0 \), square line) at various depths (\( R/D_c \)) in both half-space (hollow symbols) and edifice-loaded (solid symbols) conditions. Under half-space conditions, as a prolate reservoir becomes more elongate (i.e., as aspect ratio increases) failure is increasingly localized at the crest; this pattern is amplified when an edifice load is added. Dashed lines indicate unreasonable geometries in which the reservoir would not be expected to form because \( |P_{\text{fail}}| > |P_0| \).

3.2.2. Oblate reservoirs

Two oblate reservoirs of constant volume \( (V = 268 \) km\(^3\), equivalent to a sphere with \( R_e = 4000 \) m) are also tested, with \( a = R_a/R_b \) set to values of 0.91 \((R_e = 3750, R_a = 4131 \) m) and 0.70 \((R_e = 3160, R_a = 4500 \) m). A plot of rupture locations \((\theta_{\text{red}}, \gamma_{\text{red}})\) defined in Grosfils (2007), shown in Fig. 6, reveals rupture near the midpoint will occur for sufficiently deep oblate reservoirs; while the presence of an edifice clearly affects matters somewhat, the scaled depth of the reservoir and its aspect ratio play a more vital role in controlling the observed behavior. For more oblate reservoirs \((i.e., a = 0.7)\) failure occurs near the midpoint until, when the depth of the reservoir decreases less than \( \approx 2R_a \), the proximity of the surface shifts failure smoothly toward the crest regardless of whether or not an edifice is present; the resulting concave-up curve was also documented by Grosfils (2007) for a reservoir with \( a = 0.51 \). For an intermediate aspect ratio, \( a = 0.91 \), the behavior shown in Fig. 6 is roughly intermediate between the other cases shown, with rupture location that more closely mimics the spherical case when the reservoir is located at shallower depths then approaching the behavior observed for the more elongate \((a = 0.70)\) reservoir geometry at greater depths. We infer that the tendency for failure of an oblate reservoir to rotate uniformly toward the midpoint, as observed in the \( a = 0.70 \) case, is somewhat countered by the influence of the free surface (and edifice) on the near-spherical \((a = 0.91)\) reservoir. Taken together, these data indicate that, as a reservoir becomes increasingly oblate, the presence of an edifice generally increases the depth-dependent tendency for rupture to localize near the reservoir midpoint. As in previous figures, geologically implausible cases are noted with dashed lines.

3.3. Summary

Primary analysis of the data presented above leads to the identification of specific trends between edifice mass distribution and volume and the rupture behavior of spherical reservoirs. If the mass of an edifice of fixed volume is concentrated more toward the symmetry axis, rupture of the underlying spherical reservoir is more likely to occur at or near the crest (Fig. 2). Thus, construction of tall narrow edifices \((i.e., with stratovolcano-like geometries)\) will tend to promote failure conditions that lead to vertical dike ascent from the reservoir, a situation commonly observed at terrestrial volcanoes \((c.f. Porreca et al., 2006; Geshi, 2008; Poland et al., 2008; Soriano et al., 2008\). Conversely, if the same mass is distributed into a low, broad edifice \((i.e., a shield geometry, \( \gamma \leq 0.1 \)) the rupture behavior approaches the half-space failure conditions documented by Grosfils (2007) as \( \gamma \) decreases. When the volume ratio \( V_c/V_e \) is increased, however, the tendency for failure to
behavior, geometries, designed to extract general insight into reservoir rupture and reservoir placement and size that can occur. Under half-space conditions (Fig. 5). In contrast, oblate reservoirs or near the crest, enhancing the behavior observed for this geometry provide additional insight into patterns of rupture under elucidating the complexity of the interplay between edi... rupture location is controlled by edi... that rupture location is controlled by edi... considered to be geologically viable, as this condition must be met for our initial reservoir geometry to exist stably.

4. Discussion


For small reservoirs ($R_c = 200$ m), Pinel and Jaupart (2003) argue that rupture location is controlled by edi... their results are quite similar to those of Pinel and Jaupart (2003). Comparison between Fig. 4b i and ii reveals only minor differences, though we also quantify the magnitude of the induced rotations in iii where these data are absent in the earlier work, and note that failure will occur at the crest independent of edi... For shield geometries ($\gamma = 0.1$), however, there are considerable differences between our results and those of Pinel and Jaupart (2003). While we obtain a similar outcome for reservoirs beneath the smallest edi... significant increase in edi... eruptions observed at many shield volcanoes. Because proximity to the free surface tends to rotate...
failure away from the crest, a larger edifice is required to offset this
tendency; crudely, for the conditions modeled and \( R_c > 0 \), failure will
rotate significantly back toward the crest once \( R_c \geq \frac{3333}{(R_c/D_C) - 1000} \); this equation is simply a linear fit to the top of the grey-shaded
region depicted in Fig. 4a, part iii, for \( R_c/D_C \geq 0.3 \). In addition, if a
small reservoir forms at sufficiently shallow depths \( (R_c/D_C > 0.7) \); note
that this cannot occur unless TSL is extremely high, since a weaker
host is unable to contain the reservoir pressure at lithostatic values
without rupturing, i.e. the reservoir is considered geologically
implausible), the complex interplay between inflation-derived uplift
and conical loading stresses will shift rupture to greater depths until
sufficient edifice size is achieved and rupture shifts to the reservoir
crest, potentially promoting a prolonged era of circumferential
intrusion prior to the initiation of central eruptions.

For large reservoirs \( (R_c = 4000 \, \text{m}) \), Pinel and Jaupart (2003)
calculate that the dominant control on failure location is edifice size.
For smaller edifices \( (R_c \sim 1-2 \, \text{km}) \), the failure location is dominated by
magma reservoir inflation, with the edifice presence rotating the
failure point to slightly greater depths than in the half-space situation.
Above this 1-2 km (at most) threshold size, however, failure location
immediately shifts to the crest; this switch happens at smaller radii
for stratocones than shield volcanoes.

Our data reveal much different behavior when larger reservoirs are
present (Fig. 4c and d). First, for values above approximately
\( R_c/D_C = 0.7 \) the model geometries are physically implausible be-
cause the magnitude of the pressure required to induce failure is less
than the surrounding lithostatic stress, indicating the initial conditions
used to set up these models could not have existed in the real world.
Second, as was true for the smaller reservoir cases, when \( R_c/D_C < 0.3 \)
failure always occurs at the crest regardless of whether or not an edifice
is present; the basic reasons for this are described in detail by Grosfils
(2007). At values of the scaled reservoir depth between these two extremes, the edifice has a limited impact until achieving significant
dimensions. Similar to the smaller reservoir case discussed above,
edifices with flank slopes representative of a shield volcano must grow
above shallower reservoirs before they can begin to affect rupture
location significantly; crudely, for \( 0.3 < R_c/D_C < 0.7 \), edifice loading
effects start to have an impact once \( R_c > 10,000 \, (R_c/D_C) - 1000 \); this
equation is simply a linear fit to the top of the appropriate grey-shaded
region depicted in Fig. 4c, part iii. Still, this indicates that shield edifices
need only achieve radii of 2–6 km to lock failure at the crest. In contrast,
stratocones up to 2 km in radius (twice the size predicted by Pinel and
Jaupart, 2003) will have little effect on failure location; the grey shaded
region in Fig. 4d iii illustrates that the edifice range examined has almost
no effect on rupture location, an observation that is consistent with
the general result from Fig. 3. It is clear that smaller reservoir:edifice volume
ratios promote greater divergence from half-space failure locations
unless the reservoir is located at such great depth that it no longer reacts
significantly to the presence of a volcanic load at the surface. One
volcanological implication is that flank eruptions, and even eruptions
beyond the volcano’s margin, should continue to be quite common in
the absence of other controlling factors during the early phases of
stratocone growth.

4.1.1. Reservoir pressure and displacement

The variations between our results and those of Pinel and Jaupart
(2003) and others (e.g., Marti and Geyer, 2009) stem principally from
the two main differences in model formulation identified above: 1)
construction of the edifice as an elastic entity, and 2) incorporation
of full gravitational loading, specifically retention of the wall-parallel
components of the lithostatic stress when calculating failure location.
In order to elucidate why these differences affect reservoir deforma-
tion, we examine how net reservoir wall displacements and the
pressure required to induce failure will be interpreted under several
seemingly similar conditions (Fig. 7).

For this comparison we choose a reservoir of radius \( R_c = 4000 \, \text{m}
centered at a depth of 5 km beneath an edifice of radius \( R_e = 4000 \, \text{m} \)
and \( \gamma = 0.1 \) (Fig. 11 of Pinel and Jaupart, 2003, their conditions
\( R_e/H_e = R_c/H_c = 0.8 \)); this is an extreme example, unlikely to occur under normal
conditions (e.g., Fig. 4c iii), used here purely for illustrative purposes
when comparing with previously published results. For a fully
gravitationally loaded model and constructed edifice, we calculate
that initial failure will occur at \( \theta_{\text{fail}} = 31.1^\circ \) (Fig. 4c iii) after significant failure
of the reservoir wall occurs (location shown in Fig. 7 by a
grey circle). To reduce the fully loaded model plus an overpressure in the reservoir of magnitude \( P_{\text{fail}} = \alpha_r \) (crest) (e.g., Pinel and Jaupart, 2003), displacement of the reservoir walls is identical to
the full gravitational model, as is expected since the normal stress balance across the
wall is equivalent in the two cases. Failure to include the wall-parallel components of
the lithostatic stress in the host means, however, that the magnitude of the stresses
resulting from this displacement now exceed the TSL across the entire reservoir wall
(thick grey line), a physically implausible situation. If the overpressure value in the
“reduced” model is lowered until initial failure occurs at only one location, the resulting
displacement is much less (inner thin line) and the rupture location (partially hidden
grey semi-circle at \( r = 0 \)) shifts to the crest. These results demonstrate that failure to
employ a full gravitationally loaded model will produce either (1) a physically
implausible outcome (thick grey line), or (2) an inaccurate characterization of the
overpressure, displacement magnitude, and location required for rupture of the magma
reservoir.
the fully gravitationally loaded conditions as expected, verifying that the normal stress balance across the wall has not changed. However, by simplifying the mathematics to this commonly-employed analytical formulation, an important aspect of the physical situation is lost: the wall-parallel component of the host rock stress is eliminated in the finite element model just as it is ignored in standard analytical treatments (Grosfils, 2007). After removing this factor, one that is critical to suppressing rupture, the result is that the net displacement unrealistically causes the entire reservoir wall to fail in tension. To prevent this unrealistic outcome, if the pressure in the reduced model scenario is now dropped until failure occurs at only a single location, the wall displacement is altered significantly and the initial point of failure occurs at $\theta_{au}=0^\circ$. Fig. 7 thus demonstrates how reduction of a fully gravitationally loaded model to a traditional overpressure formulation results in inaccurate conclusions about the reservoir deformation, the rupture behavior, and the net overpressure required to induce failure, and all of these will contribute to the differences observed in Fig. 4.

4.2. Application to Summer Coon Volcano, Colorado, and Ilithyia Mons, Venus

The methodology and results presented above help establish a general conceptual framework within which the conditions promoting growth and evolution of individual edifices can be interpreted with greater clarity. To illustrate, we apply our data to two volcanoes. The first example, Summer Coon, is selected to illustrate how our model results, complementing and benefiting from field-derived constraints, can enhance existing insights into the volcanic history of a well-exposed and -studied edifice. The second example, Ilithyia Mons, is selected to demonstrate how results from our numerical modeling approach can yield insight into the history of a volcanic system even when little to no other field data or subsurface information is available; while the insights gained are obviously more speculative, this capability is of potential value to researchers attempting to understand the growth of edifices on other planets (or in poorly accessible areas of Earth) where remote sensing information is often the predominant source of data.

4.2.1. Summer Coon Volcano, Colorado

The Summer Coon stratovolcano, part of the San Juan volcanic field of Colorado, is a heavily eroded edifice. Petrological constraints ($^{40}\text{Ar}/^{39}\text{Ar}$ dates) suggest that the majority of magmatic activity at the site occurred during a span of a few hundred thousand years or less (Perry et al., 2001), implying that elastic treatment of the resultant conditions is a suitable first order method for analysis (e.g., Pinel and Jaupart, 2003). Based on field reconstructions the original edifice, assumed to be roughly conical and thought to have been emplaced on an originally horizontal surface, was 14 km in diameter with a height of around 2.2 km (Poland et al., 2008, and references therein), yielding $\gamma=0.3$. The lavas and breccias of intermediate composition that form the original cone were likely sourced from a reservoir pressure incompatible with existing petrological constraints. Since $\sigma_l$ is less than 200 MPa, this implies that the depth to the center of the reservoir is constrained to lie at a depth of $\sim 7800$ m or less (assuming magma pressure approximates the lithostatic stress, and a host rock density of $2600 \text{ kg m}^{-3}$). The reservoir would have resided significantly shallower than this, however, because at the $R/D_C$ values appropriate for tensile rupture to occur at the crest in an elastic model, the pressure $P$ within a reservoir must exceed roughly twice the magnitude of the surrounding lithostatic stress $\sigma_l$ because in a half-space $P_{\text{half/}}/\sigma_l = -3.272 (R/D_C) + 3.086 (R^2/D^2)^{0.99}$; equation derived from Fig. 10 of Grosfils, 2007). This indicates that reservoir $D_C$ was less than $\sim 3900$ m beneath Summer Coon, and the $R/D_C$ constraint simultaneously implies a maximum spherical reservoir radius of $\sim 1300$ m (volume of $\sim 9.2 \text{ km}^3$). It is worth noting that, if the volume of material released upon rupture is on the order of $\sim 1\%$ of the pre-inflation reservoir volume (cf. Blake, 1981) and the system was active for a few hundred thousand years, this implies that eruption sequences each releasing on the order of $0.09 \text{ km}^3$ occurred at an average rate of once every five to seven hundred years, a crude estimate at best but one that is generally consistent with the intervals recorded between Holocene eruptions at numerous other stratovolcanoes (e.g., Simkin and Siebert, 1994).

With the flank slope of $\gamma=0.3$ lying between the 0.1 and 0.6 cases treated previously, Fig. 4 can serve as a useful guide for assessing reservoir behavior beneath Summer Coon once the edifice had begun to form. During the main (rapid) phase of edifice growth, there are three basic possibilities: the reservoir can remain at the same scaled depth (fixed $R/D_C$); the reservoir can migrate to greater depth (which seems implausible since the system becomes more silicic and thus less dense with time, e.g, Parker et al., 2005) and/or the radius can become smaller due to changes in the net thermal balance over time (decreasing $R/D_C$); or, the reservoir $D_C$ can get shallower and/or the radius can get larger, perhaps due to assimilation of the surrounding host materials to the extent proposed by Parker et al. (2005) (increasing $R/D_C$). Assessment of these possibilities, below, indicates that the reservoir conditions likely remained largely invariant during the initial phases of Summer Coon’s construction, possibly throughout its growth, with the potential for an increase in reservoir volume once an edifice of sufficient size was in place.

In the first case, where no change in $R/D_C$ occurs, parts iii in Fig. 4 demonstrate quite clearly that failure will remain locked at the crest of the reservoir throughout the growth of the edifice, consistent with available field observations. For this outcome, while the pressure data constrain the reservoir $D_C$ to lie above $\sim 3900$ m below the surface and $R_c$ to be less than $\sim 1300$ m for reasons discussed previously, a reservoir of any size is permissible provided that it preserves an $R_c/D_C<0.3$ and is capable, upon rupture, of releasing sufficient material to explain the volumes of individual eruptions, a parameter which is not known for the main phases of edifice growth.

The second case, characterized by decreasing $R/D_C$ over time as the edifice gets constructed, is in some ways effectively a subset of the possibilities framed by the first case, as Fig. 4 parts iii illustrate that this evolutionary path will sustain conditions where rupture remains locked at the crest, promoting vertical dike ascent. As $R_c/D_C$ decreases, however, one consequence is that greater magma pressures are required to induce rupture, a restriction that will quickly render the reservoir pressure incompatible with existing petrological constraints. For example, a reservoir 1300 m across centered 3900 m beneath
the final Summer Coon edifice must achieve an internal pressure of \( \sim 176 \text{ MPa} \), or \( 1.77 \times \) lithostatic in the surrounding host rock, for failure to occur, whereas a reservoir centered at the same depth but reduced to a radius of 1000 m will require a pressure of 200 MPa (2.01 \times \) greater than lithostatic) to fail. At this depth, any further reduction in reservoir size would require pressures for rupture in excess of existing constraints. Upward migration of the reservoir (i.e., simultaneous decrease in \( D_c \)) might permit the development of smaller reservoirs without violating pressure constraints, and such a shift could be possible due to a decrease in magma density. Even if a large density contrast were to develop rapidly, however, at temperatures characteristic of such shallow depths rates of reservoir migration will be limited to perhaps a few meters per million years (Glazner, 1994). Given the brevity of the interval during which the volcanic system was active, limits to the rate at which a magma reservoir can change position within the crust, and the need to maintain a balance between reduction in reservoir size and eruption volume (linked to reservoir volume; Blake, 1981), there are practical limits to the reduction in reservoir radius that can occur without violation of independent pressure constraints. This indicates to first order that the input into, output from and geometry of the edifice-feeding reservoir likely remained in a coarse equilibrium, without significant variation, during the main episode of edifice growth.

The third general case, increasing \( R_c/D_c \), is more difficult to interpret using Fig. 4 parts iii as the outcome becomes more sensitive to the details of edifice geometry and reservoir size, but general patterns of behavior can nevertheless be inferred. For instance, momentarily treating \( R_c \) as fixed for convenience and consistency, Fig. 4a and b shows that even after only limited edifice construction has occurred rupture is locked to the crest of a small reservoir independent of the depth at which the reservoir lies, i.e. independent of \( R_c/D_c \). In contrast, for a larger reservoir, Fig. 4c and d shows that only a limited increase in \( R_c/D_c \) (to no more than 0.35) can occur without violating the constraint that rupture must remain locked at the reservoir crest, at least until the edifice reaches dimensions that are a significant percentage of the final size (roughly \( R_c = 2.5 \text{ km} \) for Fig. 4 parameters). Applying these insights, and recognizing that the flank slopes and reservoir size likely lie between the endmembers examined in Fig. 4, it appears that, until a period of time sufficient to permit significant initial growth of the edifice has elapsed, the value of \( R_c/D_c \) that characterizes the reservoir beneath Summer Coon cannot increase appreciably. The \( D_c \) could in theory shift to greater depths (up to 7800 m) to offset an increase in \( R_c \) allowing expansion of the reservoir while preserving the \( R_c/D_c \) value and maintaining the \( \sim 200 \text{ MPa} \) magma pressure constraints. This seems unlikely on physical grounds (e.g., Glazner, 1994), however, especially given the trend of the magma toward more silicic (less dense) compositions. This again suggests that the reservoir conditions sustained an approximate state of equilibrium during a significant period of Summer Coon’s development; however, once the edifice achieved a diameter approaching half the final 14 km value, the load introduced would have forced rupture of the reservoir to remain at the crest at greater and greater \( R_c/D_c \) values. This outcome would thus permit outward growth of a reservoir at fixed depth while preserving vertical dike injection, for instance if required to explain the general evolution in magma composition through ongoing assimilation fractional crystallization (AFC) processes (Parker et al., 2005). In addition, since reservoirs with greater \( R_c/D_c \) values require lower pressures to rupture, outward growth of this sort could occur without violating the constraint that magma pressure remains \( \leq 200 \text{ MPa} \).

The terminal silicic stages of activity at Summer Coon, occurring once the edifice was approaching or had effectively reached its final dimensions, are constrained by three pieces of information: (1) the magma pressures, (2) the fact that some magmas needed to erupt high enough on the edifice to occupy the fluvial channels eroded into it, and (3) the basal placement and radial orientation of the large silicic dikes.

The first constraint, magma pressure (300–600 MPa), quickly reveals that late-stage silicic activity requires the presence of a second, deeper reservoir. Whereas constraints placed on the edifice-forming eruptions indicate a maximum reservoir \( D_c = 3900 \text{ m} \), in coarse terms (i.e., neglecting the contribution added by the edifice weight) this same value is the minimum possible \( D_c \) for the silicic source reservoir because if \( R_c/D_c \rightarrow 0 \) (infinitely deep and/or small reservoir) then in half-space conditions \( P_{\text{mag}}/\sigma_c \rightarrow 3 \), implying that magma at 300 MPa can originate from a reservoir in rock at no less than \( \sim 100 \text{ MPa} \) (3900 m). The maximum depth, to match lithostatic conditions of 300 MPa in the surrounding rock, is \( \sim 11.8 \text{ km} \). By the same reasoning, the minimum depth for a reservoir hosting magma at 600 MPa will be roughly 7800 m, while the maximum possible depth for such a reservoir is that at which the lithostatic pressure matches the magma pressure, or \( \sim 23.5 \text{ km} \). If multiple reservoirs are involved in the late-stage silicic activity, the pressure constraints simply indicate that they would have to form between depths of 3900 m and 23.5 km. A single reservoir can readily explain all of the late-stage activity, however, provided that it formed and evolved at depths greater than 7800 m and shallower than 11.8 km.

The second constraint, having magma capable of eruption near the edifice crest, requires that two elements be achieved. First, magma from the underlying reservoir must move upward when rupture occurs; this condition is not difficult to meet, since Fig. 4 shows that, when \( R_c = 7000 \text{ m} \), rupture will remain locked at the crest of the reservoir for any physically plausible value of \( R_c/D_c \). Second, the stress conditions within the edifice must help promote movement of magma via dikes to the upper portions of the volcano; this, also, is not difficult to achieve, as we show below.

The third constraint is that emplacement of a radial dike system must be favored by stress conditions near the base of the edifice. Our models do not incorporate factors such as structural or rheological effects (e.g., Rubin, 1995; Dahm, 2000; Gudmundsson, 2003; Gudmundsson and Loetveit, 2005; Gudmundsson, 2006; Gudmundsson and Philipp, 2006), the dynamic stresses associated with dike propagation (cf. Meriaux and Lister, 2002), etc., and are thus a poor basis for predicting dike paths and why one route would be favored over another at any given time. Nonetheless, in general terms—and assuming that dikes follow a path perpendicular to the least compressive stress orientation—our models produce stress conditions that match the field observations at Summer Coon remarkably well. For instance, consider a reservoir centered at a depth of 9800 m, in the middle of the 7800 m to 11.8 km range permitted (coarsely) for a single silicic reservoir to match the pressure constraints placed on the late-stage silicic activity. With a reservoir radius of 2000 m, rupture occurs at the crest when \( P_{\text{mag}} = 579 \text{ MPa} \) (i.e., \( \sim 600 \text{ MPa} \)), and Fig. 8 illustrates the associated stress state in the edifice and shallow substrate. The resultant vertical intrusion will have a radial alignment as it ascends. When it reaches the base of the edifice, three outcomes are possible. If the intersection occurs near the axis of the volcano (path a), the stress state is consistent with continued vertical ascent and hence eruption near or at the crest of the volcano. If the dike intersects the base of the volcano further out and penetrates into it to a significant degree (path b), the host rock stress conditions will focus the intrusion inward creating an outward-dipping conical feature. This inward focusing will thus tend to gather material toward the central axis near the base of the volcano. A sharp transition like this from radial dike to conical sheet will be difficult to achieve in practice, but the net effect should be to inhibit dike penetration into the edifice away from the central axis, and we propose that any inward focusing and collecting of material that occurs may help explain the formation of the central intrusive suite documented at Summer Coon. If correct this would suggest that the central intrusive suite is a passive outcome of near-vertical dike propagation rather than the crest of a shallow reservoir system that was integrally involved in the formation and evolution of the edifice as suggested by Parker et al. (2005); the shallow reservoir proposal is inconsistent with our model results. Finally, if
penetration into the edifice does not occur, or occurs to only a limited extent consistent with observed dike segmentation, the stress conditions favor formation of radial intrusions along the basal contact of the edifice (path c). This radial geometry is favored across nearly the entire radius of the volcano, and we note as well that calculated differential stress magnitudes in the host decrease steadily outward from $\sim 40$ MPa near the axis to $\sim 10$ MPa near the edge of the edifice; these model results are closely consistent with the lengths and outward thickening characteristic of the silicic dikes at Summer Coon (Poland et al., 2008). We note that the stress state beneath the volcano does not preclude simple vertical ascent of dikes with a planar or fan-shaped geometry, but it is equally consistent with central ascent and then shallow re-alignment of dikes with a blade-like geometry; the latter hypothesis, supported by field observations indicating a predominance of lateral flow plus thermal considerations, is more heavily favored at Summer Coon and is consistent as well with observations at other stratovolcanoes (e.g., Poland et al., 2008; Acocella and Neri, 2009).

4.2.2. Ilithyia Mons, Venus

Ilithyia Mons, a large conical volcano on Venus exhibiting a shield morphology ($H_e = 625$ m, $R_e = 54.5$ km), is centered at $13^\circ$S $315^\circ$E (Fig. 9). A series of centrally-derived edifice flows defines the current surface of the edifice, and late-stage, circular caldera collapse events at the crest, ranging in radius from 1.5–8 km, lead us to adopt a plan view reservoir radius of 4 km (based on caldera-to-reservoir size relationships, e.g., Marti et al., 1994). A distinctly circumferential lineament that partially encloses the central caldera cluster on the west and south sides, with an approximate radius of 18 km, is the only other major structure cross-cutting the edifice flows, though its origin...
is poorly constrained. The edifice is partially covered by late-stage regional plains volcanism, and we note that like Ilithyia Mons neither the plains units predating the current surface of the edifice nor those plains units which postdate the edifice record significant tectonic deformation, consistent with our model conditions since regional differential stress effects appear to have been negligible during the period of edifice growth. As noted previously, our goal is to illustrate the potential value of our modeling method and results when applied to a volcano for which only interpretations derived from remote sensing data are available or possible; the specific details described below should thus not be taken as a unique characterization of the subsurface geometry associated with Ilithyia Mons. While clearly a sharp contrast to the situation for Summer Coon, this ‘limited data’ scenario remains a common circumstance for volcanoes in remote areas of Earth, and is certainly the interpretive challenge faced by planetary volcanologists.

Some constraints upon reservoir placement can be inferred from the history of late-stage central eruptions. A spherical reservoir of radius \( R_c = 4000 \text{ m} \) beneath a flat surface, required to fail at the crest, would have to lie at a \( D/C \) of \(-1.14 \text{ km} \) \((\alpha = 1.0, \text{ no edifice}^\circ \text{line of Fig. 6})\) or greater in order to yield the vertical eruptions required to initiate the growth of Ilithyia Mons. As the edifice grows and reaches its current size, rupture would continue to produce vertically ascending dikes for spherical reservoirs as shallow as \( D/c = 8.5 \text{ km} \) \((R_c/D_c = 0.47; \text{ Fig. 6})\). Given the range in calderas sizes observed, however, there is some uncertainty about the size of the late-stage reservoir; a reservoir of radius 1.5 km could be centered as little as 3.2 km below the surface, generally consistent with models of shallow reservoir formation in areas of Earth, and is certainly the interpretive challenge faced by planetary volcanologists.

The one remaining piece of information available to us at Ilithyia Mons is the arcuate fracture, lying at an elevation 450 m above the plains units which postdate the edifice growth. As noted previously, our goal is to illustrate how variations in host rock density structure have no appreciable effect on rupture location \((\text{Grosfils, 2007})\), a result that appears to hold for edifice-loaded conditions based on a limited number of tests. If this is correct and a smoothly-varying (pore space compaction) configuration is assumed for the subsurface density structure \((\text{as in Head and Wilson, 1992})\), then the reservoir depths in excess of \(-3 \text{ km} \) predicted above imply magma densities of roughly \(-2700 \text{ kg m}^{-3} \) or greater \((-2800 \text{ kg m}^{-3} \text{ at } -15 \text{ km}) \) for neutral buoyancy conditions, compatible with the mafic compositions believed to characterize the surface of Venus.

The remaining piece of information available to us at Ilithyia Mons is the arcuate fracture, lying at an elevation 450 m above the edifice-surrounding plains at a radius of \(-18 \text{ km} \) from the summit. For illustrative purposes we test the hypothesis that this lineament is volcanic \((i.e., \text{a response to shallow circumferential intrusion}) \) rather than purely structural \((i.e., \text{due to flexure or some other process})\), and evaluate whether this lineament, if volcanic, can be used to better constrain the nature of the subsurface magmatic evolution at Ilithyia Mons.

We begin by considering several volcanic scenarios that could explain the formation of the observed arcuate feature. First, we keep the spherical reservoir radius fixed and evaluate whether vertical migration of the reservoir \((\text{and associated rotation of the point of failure away from the crest})\) could yield the observed arcuate feature, a migration akin to that invoked for instance by \text{Glazner and Miller (1997)} in response to magma evolution and shifts that preserve neutral buoyancy; in contrast to the case at Summer Coon, where thermal and temporal constraints severely limit vertical migration processes, we must allow for this possibility at Ilithyia Mons since the absolute duration of the volcanic activity remains unknown. Fig. 6 illustrates that a spherical reservoir will have to lie at depths shallower than 8.5 km for failure to shift away from the crest, and that the location will never rotate away by more than \(-25^\circ \) \((i.e., \theta > 25^\circ) \) with the edifice present. If we assume, to obtain an initial back-of-the-envelope estimate of reservoir placement, that an intrusion initiating at the reservoir essentially follows a straight path to the surface \((\text{static half-space models of simple and complex configurations generally show gentle steepening upward; e.g. Gudmundsson, 1998, Klausen, 2004, Gudmundsson and Philipp, 2006})\), then to initiate a circumferential intrusion that intersects the surface at the location of the arcuate lineament, with \( R_c = 18 \text{ km} \) and \( H_c = 450 \text{ m} \) requires a combination of \( \theta \) and \( D/C \) that meets the condition

\[
D_i/C = (R_i / \tan \theta) - H_i.
\]

For \( D_i/C < 8.5 \text{ km} \), solving Eq. (2) indicates that the associated \( \theta \) values required will never be less than about \(65^\circ \). Thus, the observed sequence of vertical eruptions followed by a late-stage arcuate intrusion cannot be achieved through vertical migration of a spherical reservoir.

Another possibility motivated by observed behavior in terrestrial systems derives from variations in the temperature or in the volumetric rate at which magma is fed into the reservoir from sources at greater depth \((\text{e.g. Muller and Pollard, 1977; Turner and Campbell, 1986; Klein et al., 1987; Greenough and Hodych, 1990})\). For instance, thermal cooling and size reduction during a quiescent phase could be followed by re-supply and pressurization of the smaller reservoir. Fig. 6 shows clearly, however, that reduction in the size of a spherical reservoir at a given depth cannot yield failure farther from the crest. Alternatively, thermally-induced changes in the rheology of the rocks surrounding a long-lived reservoir that fed material to the surface via central eruptions can lead to a greater effective elastic size \((\text{e.g., Jellinek and DePaolo, 2003})\). If the spherical reservoir’s \( D/C \) remains constant while the radius \( R_c \) is increased, Fig. 6 and Eq. (2) also suggest that this type of variation will not be capable of creating the circumferential intrusion required; explicit modeling of these radius changes \((\text{necessary because Fig. 3 illustrates that failure location is also sensitive to \( V_r/V_c \) provides quantitative verification of this back-of-the-envelope answer, revealing that rupture angles never rotate away from the crest by more than \(-30^\circ \), far short of the \(65^\circ \) or more required to create the arcuate lineament}.

Third, vertical delivery of magma from a spherical reservoir at great depth \((\text{i.e. initial} D_i/C > 11.4 \text{ km})\) might at some stage during the growth of the edifice create a shallower secondary reservoir \((\text{e.g., as discussed by Sigurdsson et al. (2000)}\) and proposed by \text{Marti and Geyer (2009)} for Teide-Pico Viejo in the Canary Islands\) which, when subsequently pressurized to the point of failure, then creates the intrusion geometry required to generate the larger arcuate lineament. Spherical reservoirs will not make good candidates for this secondary reservoir, for reasons discussed above, and prolate reservoirs simply enhance the tendency for failure to localize at the crest and so are equally unsuitable \((\text{Fig. 5})\). Fig. 6 indicates that a shallow oblate reservoir might be suitable; however, in this case our crude assumption of linear propagation normal to the reservoir wall from the point of failure requires that a successful geometry must meet the condition

\[
D_i/C = (R_i - R_c \cos \alpha)(\tan \pi / 2 - |\tan^{-1}(Y_t / X_t)|) - H_i + R_c \sin \alpha.
\]

where

\[
X_t = -R_c \sin \alpha / (R_c^2 \cos^2 \alpha + R_c^2 \sin^2 \alpha)^{0.5}
\]

\[
Y_t = R_c \cos \alpha / (R_c^2 \cos^2 \alpha + R_c^2 \sin^2 \alpha)^{0.5}
\]

\[
\text{are the components of the unit vector tangent to the ellipsoid at angle} \alpha = 90^\circ - \theta. \text{If a volume equivalent to a spherical reservoir with} \ R_c = 4000 \text{ m} \text{is retained, an oblate reservoir of aspect ratio 0.91 would}
\]
require $D_C \approx 13 \text{ km}$ to form the arcuate lineament, necessitating a spherical reservoir at even greater depth as the feeder, while an oblate reservoir with an aspect ratio of 0.70 would reside at a $D_C \approx 6.2 \text{ km}$, necessitating a feeder reservoir (if spherical) at only slightly greater depth than the minimum allowed to promote vertical eruption. These solutions, in each case requiring a plan view oblate ellipsoid radius consistent with the observed surface caldera sizes, are of course non-unique, simply drawing upon our tested examples (i.e., Fig. 6) for convenience. Given the $V_c/V_e$ sensitivity, other examples need to be modeled explicitly; for example an oblate ellipsoid with a volume half that of the examples discussed above and an aspect ratio of 0.70 could generate the intrusion necessary to match the arcuate feature if it resides at a depth of $\sim 5 \text{ km}$.

The back-of-the-envelope approach used above suggests that the arcuate lineament observed, if volcanic in origin, could only originate from an oblate reservoir, the depth of which is determined by reservoir size and aspect ratio. When the oblate systems proposed above are modeled explicitly, however, it quickly becomes clear that the stress state within the host rock and edifice are modeled explicitly, however, it quickly becomes clear that the physical situation, in traditional fashion, to an overpressurized sphere in an elastic, gravity-free host beneath an imposed edifice load. The principal differences in the modeled rupture behavior, summarized in Fig. 4 and illustrated in Fig. 7, appear to occur because (1) we include key wall-parallel stress components ignored in previous models, the absence of which affects (a) interpretation of reservoir wall displacement, (b) the pressure required to induce initial failure, and (c) initial rupture location, and (2) we incorporate the edifice as a 'physical' element within our model, one capable therefore of elastic response as well as applying a vertical load to the upper surface of the half-space.

When compared to half-space conditions, the weight of an edifice generally shifts the rupture location towards the crest of a spherical reservoir. The induced changes in failure location are often quite small (Fig. 4), but in general a large edifice has a more significant effect on rupture re-location, with rupture approaching the reservoir crest as the edifice grows steeper (Fig. 2) or larger (Figs. 3 and 4). A large reservoir generally requires much larger edifices to induce this shift in rupture location to the reservoir crest than a small reservoir. Large reservoirs at shallow depths are often not geologically plausible because the pressure required to induce rupture is less than the surrounding lithostatic pressure ($|P_{lith}| - |P_{amb}|$, a specification which dictates that a stable reservoir (i.e., the initial model geometry defined) could not have formed under these conditions.

When non-spherical reservoirs are included in our models, the presence of an edifice generally amplifies observed half-space rupture behavior (Figs. 5 and 6). For prolate reservoirs, rupture occurs at the

5. Conclusions

In this study we investigate how the presence of an edifice affects rupture location and orientation for inflating reservoirs of various geometries. Full gravitational models, implemented using finite element techniques, yield results different from previous studies (e.g., Pinel and Jaupart, 2003; Marti and Geyer, 2009) that reduce the physical situation, in traditional fashion, to an overpressurized sphere in an elastic, gravity-free host beneath an imposed edifice load. The principal differences in the modeled rupture behavior, summarized in Fig. 4 and illustrated in Fig. 7, appear to occur because (1) we include key wall-parallel stress components ignored in previous models, the absence of which affects (a) interpretation of reservoir wall displacement, (b) the pressure required to induce initial failure, and (c) initial rupture location, and (2) we incorporate the edifice as a 'physical' element within our model, one capable therefore of elastic response as well as applying a vertical load to the upper surface of the half-space.

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![Fig. 10. Idealized cross section through the central portion of the Iolithyia Mons edifice showing a shallow oblate reservoir ($R_i = 2500 \text{ m}, R_z = 3580 \text{ m}, D_C = 5000 \text{ m}$). The grid of symbols, with 250 m spacing, shows local alignment of the least principal stress when the reservoir is pressurized to the point of rupture; dots thus indicate conditions suitable for radial dikes and lateral sills. Path (a) shows the idealized ‘straight propagation path’ solution required to explain the circumferential lineament at $r = 18 \text{ km}$ (located at star; see text for details), while path (b) shows schematically how the circumferential intrusion originating at the point of failure should transition to a radially aligned dike at some distance from the reservoir. As is true for the example solution shown, we identified no conditions under which the circumferential lineament is likely to originate as the surface expression of a subsurface intrusion, and the absence of a radial fracture system on the edifice flanks demonstrates that at least the later stages of the edifice’s volcanic history were characterized predominantly by centralized summit eruptions.](image)
reservoir crest for shallower reservoirs, never leaving the crest for more elongate reservoirs of large aspect ratios (e.g., \(a = 1.4\)). For oblate reservoirs, rupture occurs closer to the reservoir mid-depth as aspect ratio decreases.

In all cases examined (spherical, prolate, oblate; with and without an edifice) initial rupture occurs in the orientation of \(c\), near the point of maximum strain deviation, where tensile stresses induced by reservoir inflation first exceed the tensile strength of the host rock. Depending on the exact rupture location, this promotes initial formation of vertically ascending dikes (of any axial orientation when failure occurs at the crest), lateral sills, or circumferential intrusions, but under no circumstances examined is initial lateral emplacement of radial dikes favored. These results are directly analogous to those of G罗斯fils (2007).

Our methods and results, in addition to yielding general non-dimensionalized insight into edifice–reservoir interactions, can be combined with surface stratigraphy or other geological constraints (e.g., knowledge of eruption volumes or pressures; geophysical constraints on magma reservoir size or depth; intrusion behavior) to assess the magma reservoir characteristics (e.g., radius, depth, geometry) and evolution for specific volcanoes. At Summer Coon for example, a well-studied stratocone on Earth, we show how our analysis: (a) augments existing studies by constraining reservoir depth \(D = 3900\) m and size \((R_c = 1300\) m) prior to edifice formation; (b) allows us to assess possible paths of reservoir evolution during the main phase of edifice growth (the reservoir largely retains the initial geometry, but is capable of a significant size increase once the edifice becomes sufficiently large); and, (c) helps demonstrate mechanically that, unless multiple reservoirs somehow formed quite rapidly, late-stage silicic activity likely originated from a single reservoir between the depths of \(8–12\) km, with stress states derived from reservoir inflation and rupture beneath Summer Coon consistent with the observed characteristics of a radial dike swarm emplaced near the base of the edifice (e.g., Poland et al., 2008). In contrast, at Lithya Mons on Venus, a stratigraphically simple shield-like edifice that preserves a history of central eruptions and summit caldera collapse, our models suggest a straightforward volcanic history involving only pressurization and rupture of a spherical (or prolate) reservoir centered at a depth of at least \(3\) km. In addition, based on our model results we can effectively eliminate reservoir-derived volcanic intrusion as the cause of a circumferential lineament located on Lithya Mons’ flank. While these simple interpretations for Lithya Mons are by default limited due to the paucity of independent observations, they nevertheless represent an important step forward in our understanding of the volcanicological conditions that existed at the time the edifice was forming. As demonstrated at Summer Coon, further insights into the volcanicological history of Lithya Mons and other understudied edifices can be gained from our models if/as other data (i.e., individual eruption volumes) are obtained.

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