Vertical Movement in Mare Basins: Relation to Mare Emplacement, Basin Tectonics, and Lunar Thermal History

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The spatial and temporal relationships of linear rilles and mare ridges in the Serenitatis basin region of the moon are explained by a combination of lithospheric flexure in response to basin loading by basalt fill and a time-dependent global stress due to the thermal evolution of the lunar interior. The pertinent tectonic observations are the radial distance of basin concentric rilles or graben from the mare center, the location and orientation of mare ridges, interpreted as compressive features; and the restriction of graben formation to times older than 3.6 ± 0.2 b.y. ago, while ridge formation continued after emplacement of the youngest mare basalt unit (~3 b.y. ago). The locations of the graben are consistent with the geometry of the mare basalt load expected from the dimensions of multiring basins for values of the thickness of the elastic lithosphere beneath Serenitatis in the range 25-50 km at 3.6-3.8 b.y. ago. The locations and orientations of mare ridges are consistent with the load inferred from surface mapping and subsurface radar reflections for values of the elastic lithosphere thickness near 100 km at 3.0-3.4 b.y. ago. The thickening of the lithosphere beneath a major basin during the evolution of mare volcanism is thus clearly evident in the tectonics. The cessation of rille formation and the prolonged period of ridge formation are attributed to a change in the global horizontal thermal stress from extension to compression as the moon shifted from net expansion to overall cooling and contraction. Restricting the time of peak lunar volume to 3.6 b.y. or earlier, together with the constraint that the accumulated thermal stress since the end of heavy bombardment did not reach levels capable of causing global-scale lithospheric failure, greatly limits the range of possible lunar thermal histories. The zone of horizontal extensional stresses peripheral to mare loads favors the edge of mare basins as the preferred sites for mare basalt magma eruption in the later stages of mare fill, although subsidence may lead to accumulation of such young lavas in basin centers.

INTRODUCTION

The tectonics on a planet's surface are the product of lithospheric stresses generated by processes on both global and local scales. Global processes include changes in planetary spin rate and changes in planetary volume due to thermal evolution and differentiation of the interior. Local processes include the formation of impact craters and basins and the loading of the lithosphere from above by volcanic or sedimentary deposits. The stress regime in the lithosphere, on either a global or a local scale, also exerts a control on volcanism, with horizontal extensional deviatoric stress tending to favor volcanism and horizontal compressive deviatoric stress tending to shut volcanism off.

The moon shows little or no direct evidence in its surface tectonics for global stresses large enough to have caused widespread lithospheric failure. There are no huge chasms as there are on Mars nor is there a pervasive system of large thrust or reverse faults as there is on Mercury. This observation restricts the magnitude of stresses generated by global processes to less than the strength of typical lunar crustal material. In particular, the constraint that the thermal stress associated with lunar thermal history has not generally fractured the lithosphere in the period since heavy bombardment greatly restricts the moon's possible evolutionary tracks. Early extensive heating and melting of the outer 200–300 km of the moon and an initially cold deep interior are favored [Solomon and Chaiken, 1976; Solomon, 1977].

Most of the tectonic features on the lunar surface are local in scale and are associated with vertical movement in and near mare basins. These features include the linear and arcuate rilles, generally interpreted as graben and most commonly peripheral to major mare basins, and the mare ridges, most of which are localized within mare units and are plausibly identified as compressive features indicative of crustal shortening. The spatial distribution of rilles and ridges, the sequence and distribution of major volcanic units, and the topography of the present surface for most maria, including those with mascon gravity anomalies, are all evidence for an extended period of subsidence of mare basins spanning most of the era of mare volcanism and perhaps continuing until long after volcanism ceased [Howard et al., 1973; Muehlberger, 1974].

The distribution in time of rille and ridge formation, however, is far from uniform. Linear rilles occur only in relatively old mare units and in highlands generally adjacent to maria and appear to have all formed prior to 3.6 ± 0.2 b.y. ago [Lucchitta and Watkins, 1978]. Mare ridges, on the other hand, cut even the youngest mare basalt deposits, and therefore these features must have continued to form until times younger than 3.0 b.y. ago.

We develop in this paper the hypothesis that the spatial and temporal distributions of linear rilles and mare ridges in the mare regions of the moon are the products of two superposed stress systems (Figure 1): a local stress due to lithospheric loading by basalt fill in the mare basin and a global thermal stress due to the thermal history of the lunar interior. The local horizontal stress radial to the mare basin is compressive beneath the load and extensional at the edge of the load (Figure 1, top). Under regional extension the extensional stress in the...
outer portions and outside of the mare units is enhanced (Figure 1, middle). Under regional compression the zone of peripheral extension is suppressed, and the central compressive stresses are increased (Figure 1, bottom). For a subset of the lunar thermal history models that successfully accounts for the lack of global-scale extensional or compressive tectonic features an early period of modest global expansion and lithospheric extension is confined to the first 1.0 b.y. of lunar history and is followed by a more extended period of modest planetary contraction and lithospheric compression lasting until the present. These models account for cessation of linear rille formation by the onset and gradual increase of global horizontal compressive stress associated with the period of lunar contraction. The hypothesis of two scales of stress (local and global) is extended to incorporate the volcanic and loading history of the mare basins and the timing and eruption sites of the youngest mare basalt flows.

In the next section the evidence is assembled on the volcanic and the tectonic history of a well-documented type case for mare basin evolution, Mare Serenitatis. The probably pre-volcanic basin structure, the age sequence and relative volumes of major surface units, and the spatial and temporal relationships of rilles and ridges to volcanic units and to the subsidence history are all considered.

In the following section we detail a quantitative model for the local stress in the Serenitatis basin region, utilizing flexure theory for a fluid-filled elastic shell. The model is based on the most likely geometry for mare basalt fill, using the Orientale basin as an analog, and on the excess mass determined from mascon gravity analysis. We show that the spatial distribution of graben and ridges, as well as the history of subsidence of the Serenitatis basin inferred from subsurface mapping of basalt units by radar sounder data and from the present mare surface topography, matches the predictions of the local model for a suitable choice of the elastic parameters of the lithosphere. In particular, there is strong evidence for an increase in the effective thickness of the elastic lithosphere between the time ~0.1 b.y. after basin formation and the time when mare volcanic filling ceased.

We next discuss the global thermal stress that is a predictable consequence of planetary thermal evolution in the context of both the class of models that account for the absence of global-scale tectonics and the specific and quantitative models that give lithospheric extension only in the first 1.0 b.y. of lunar history. The superposition of the local and the global stress fields then provides a simple explanation for the time dependence of rille and ridge formation. This explanation is consistent with only a very limited suite of possible lunar thermal histories.

The dual-scale stress model for mare basin tectonics is extended to a conceptualized description of the evolution with time of both the volcanic load and the response of the lithosphere to that load. The hypothesis that mare volcanism is favored by horizontal extensional stress in the lithosphere leads to the predictions that the eruption sites for the youngest mare basalt flows should generally be localized at the edges of mare basins and that mare volcanism tended to shut off when global compressive stress finally overrode the extensional bending stresses associated with mare basalt loading. We conclude with some suggestions for the application of the ideas in this paper to detailed studies of other lunar basins and of large basins on other terrestrial planets.

THE SERENITATIS BASIN EXAMPLE

The Serenitatis basin is of impact origin and has subsequently been filled with lavas, producing Mare Serenitatis, approximately 600 km in diameter. Unlike the younger and more well-preserved Imbrium and Orientale examples, Serenitatis basin-related topography and deposit texture are poorly preserved, and correlation of features with the younger basins has proved difficult. In particular, the close proximity of the younger Imbrium basin has led to extensive modification of Serenitatis basin features and controversy over the age of the basin and the location of its rings.

Ring Structure. The large number of basins on the moon means that each succeeding basin is likely to overlap previous ones and be influenced by previous topography and structure. Serenitatis is no exception. In addition to Tranquilitatis to the south, Scott [1974] documented a smaller basin north-northwest of the Mare Serenitatis region (Figure 2) and suggested that its presence explained the irregular mass distribution within Serenitatis. Wolfe and Reed [1976] and Reed and Wolfe [1975] describe the general geometry of the two basins and designate them the northern and southern Serenitatis basins. Although it is not specifically stated, northern Serenitatis appears to predate the southern Serenitatis event. In this study we concentrate on the southern Serenitatis basin and refer to it as Serenitatis.

A recent reconstruction of the Serenitatis basin structure [Head, 1979] (Figure 2b) is based on detailed mapping of the Serenitatis region and comparisons to Orientale and other large craters and basins. Craters larger than about 150 km are characterized by additional rings concentric to the crater rim [Hartmann and Wood, 1971; Head, 1974b; Wood and Head, 1976]. The innermost observed ring in Serenitatis is indicated by the distinctive concentric ring of mare ridges developed on mare deposits (radius of 200 km). Reed and Wolfe [1975] proposed that this ring corresponds to the edge of the inner depression in Orientale. Head [1979], however, correlated this ring with the Orientale peak ring or Inner Rook Mountain ring on the basis of ring spacing and the association of the peak ring and the ridge ring in Imbrium.

Wilhelms and McCauley [1971] connected the Haemus Mountains and the Taurus-Littrow massif (Apollo 17) to form the second ring. Wolfe and Reed [1976], on the other hand,
make the Haemus the second ring and draw a separate third ring through the Apollo 17 site (Figure 2a). On the basis of the location of the first major topographic break, symmetry with other rings, ring spacing, and ring morphology, Head [1979] placed the second ring (analogous to the Outer Rook Mountain ring in Orientale) along the Haemus Mountains and at the western edge of the eastern basin highlands (Figure 2b). The third ring has previously been drawn in several places (Figure 2a). The identification shown in Figure 2b is drawn on the basis of the location of a major topographic break and scarps visible under low lighting conditions and in topographic maps. Distribution of domical facies and ring spacing are cited as additional evidence for the correlation of this third ring with the Cordillera ring in Orientale. In this paper we adopt the ring assignments and correlations of Head [1979] (Figure 2b, Table 1).

**Basin Age.** Serenitatis has been placed in a relatively old position in the sequence of nearside basins (predating Orientale, Imbrium, Crisium, Humorum, and Nectaris) on the basis of morphologic degradation of basin features and number of superposed craters [Stuart-Alexander and Howard, 1970]. Harmann and Wood [1971] classified Serenitatis as being very old in their basin sequence (which differs from that of Stuart-Alexander and Howard). Although Serenitatis seems very old, the effects of the Imbrium event (ejecta and seismic shaking) may have increased the apparent relative age of Serenitatis. A recent analysis of the far field ejecta (secondary craters) of basins has led to reinterpretation of basin sequences and placement of Serenitatis younger than Nectaris and perhaps Humorum [Wilhelms, 1976].

Breccias collected from boulders at the Apollo 17 site have been interpreted as Serenitatis ejecta. Recent analysis of 73215 by the James Consortium [James and Blanchard, 1976] shows that many breccia components were not isotopically equilibrated (i.e., reset to the age of the event) during breccia formation [Jessburger et al., 1977; Müller et al., 1977]. Thus the youngest ages of matrix and clast samples are still upper limits to the assemblage event. A contained felsite clast in 73215 is thought to be an exception, however, on the basis of identical ages for crystalline and glassy components in the clast [Jessburger et al., 1977], the demonstration that the glass component was a melt when the clast was incorporated in the breccia [James and Hammerstrom, 1977], and the lack of evidence for later reheating [Nord and James, 1977]. From the ages of the felsite clast (with revised decay constants and potassium isotopic ratio) and of three vesicular aphanitic matrix samples from the closely related breccia 73255, Staudacher et al. [1978] conclude that the Serenitatis basin-forming event occurred at 3.87 + 0.04 b.y. We adopt this value in the discussion below.

**Basin Geometry.** The well-preserved, relatively unflooded Orientale basin is generally comparable in size with Serenitatis (Table 1) and provides information on the initial basin geometry and early flooding style. Orientale geometry (Figure 3a) is represented by basalts presently exposed as part of an annulus along the eastern and southwestern parts of the basin (Figure 4). These lavas flood graben developed in the earlier present topography, which includes layers of impact melt deposits [Head, 1974b; Moore et al., 1974] and mare basalt [Head, 1974b; Greeley, 1976]. In the inner depression of Orientale, combined thicknesses of these two units probably do not exceed 2 km. An additional factor for eastern Serenitatis is the emplacement of ejecta from the Imbrium basin which will serve to erode basin floor topography and pond the combined ejecta and eroded debris in low-lying areas. Estimates of the variation in thickness of the Imbrium ejecta contribution (assuming radial geometry) range from about 600 m closest to Imbrium to about 100 m at the Apollo 17 site [McGetchin et al., 1973]. Even if these numbers are correct (which is unlikely because of the number of simplifying assumptions and the extrapolation to basin size impacts), there remain major uncertainties concerning the mode of emplacement of ejecta and its ultimate resting place. We conclude that there is a breccia component within the basin due to the Imbrium event, that it may be highly irregular in thickness but generally thins from west to east, that its thickness never exceeds about 1 km, and that its emplacement may have served to smooth Serenitatis floor topography by both erosion and infilling. Emplacement of Imbrium ejecta certainly disturbed the coherent impact melt sheet covering the inner basin. On the basis of the above discussion, we adopt an appropriately scaled Orientale basin geometry for the unflooded Serenitatis basin (Figure 3b), and we do not model either the impact melt sheet [Head, 1974b; Hörz, 1978] or the Orientale basin ejecta component.

**Volcanic and Tectonic History.** The present surface distribution of mare units has been mapped by Thompson et al. [1973] and Howard et al. [1973]. Detailed studies of mare surfaces invariably suggest a more complex picture for detailed surface unit distribution [Johnson et al., 1975] and ages [Boyce, 1976]. We follow the generalized stratigraphy of Howard et al. [1973] and supplement it where appropriate and necessary (see Figure 4, caption). Earliest basalt deposits of Serenitatis (Figure 4, unit I) occur today in a belt around the southern rim, continuous with Mare Tranquillitatis and including the Apollo 17 site. Ages of basalt samples from the site fall within the range 3.65-3.85 b.y. [Wasserburg et al., 1977; Geiss et al., 1977], and the dark mantle soil component appears to be contemporaneous (~3.7 b.y.) [Huassin and Schaeffer, 1973]. Subsequent to their emplacement, these basalts were downwarped toward the basin center, and a series of basin concentric graben formed in these units and adjacent highlands along the edges of Serenitatis. A second stage of filling (unit II) is represented by basalts presently exposed as part of an annulus along the eastern and southwestern parts of the basin (Figure 4). These lavas flood graben developed in the earlier

### TABLE 1. Ring Assignments and Diameters for the Serenitatis Basin

<table>
<thead>
<tr>
<th>Diameter, km</th>
<th>Orientale Equivalent, km</th>
<th>Diam-eter, km</th>
<th>Orientale Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inner ring</strong></td>
<td>400</td>
<td>Inner depression (edge of mare), 320</td>
<td>420</td>
</tr>
<tr>
<td><strong>Second ring</strong></td>
<td>580</td>
<td>Inner Rook, 480</td>
<td>610</td>
</tr>
<tr>
<td><strong>Third ring</strong></td>
<td>750</td>
<td>Outer Rook, 600</td>
<td>880</td>
</tr>
<tr>
<td><strong>Fourth ring</strong></td>
<td>1300</td>
<td>Cordillera, 900</td>
<td>not recognized</td>
</tr>
</tbody>
</table>

From Reed and Wolfe [1975] and Wolfe and Reed [1976] and From Head [1979]
Fig. 2a

Fig. 2. Serenitatis basin ring structure. Shown are (a) previous ring assignments (dashed lines are from Wilhelms and McCauley [1971], obliquely dashed lines are from Wolfe and Reed [1976]) and (b) ring assignments used in this study. The inner ring is interpreted as the peak ring (Inner Rook equivalent in Orientale), the second is equivalent to Outer Rook, and the third to Cordillera [from Head, 1978]. Arcuate structure in the northwest is a remnant ring from the older northern Serenitatis basin. Solid stars indicate locations of Apollo 15 and 17 sites. Base map is NASA LMP-I; 1° of latitude is about 30 km.

units and are mapped as Eratosthenian-Imbrian in age (Figure 5), although locally patches may be younger. The unit II lavas have been substantially downwarped toward the basin center but contain only minor fractures and graben. The central downwarped part of Serenitatis was filled with a third stage of lavas (unit III) which postdate virtually all graben formation. More ridges are developed extensively in this unit and appear to be compressional features [Howard et al., 1973; Howard and Muehlberger, 1973; Muehlberger, 1974; Head, 1974a; Maxwell et al., 1975; Lucchitta, 1977] associated with downwarping. The deformed surface indicates continued downwarping well past emplacement of the youngest lavas in Serenitatis. The central part of Serenitatis is now generally low relative to the rest of the mare but also contains a slight topographic high at present. Mare ridges in unit III show as much as 300- to 450-m elevation relative to surrounding parts of unit III, suggesting that at least this much relative deformation occurred during and subsequent to the emplacement of unit III.

Ages for the different stages of filling are difficult to determine because of lack of returned samples. On the basis of Apollo 11 and 17 ages, unit I covers a range of about 3.65-3.85 b.y. In terms of crater counting results, unit III appears homogeneous and is similar to the Apollo 15 landing site [Neukum et al., 1975]. Crater degradation ages appear variable [Boyce, 1976], but much of the area of unit III is ~3.4 b.y. old. On the basis of incomplete and somewhat conflicting data, we
assign an age to unit III of 3.0-3.4 b.y. Several younger patches of mare (~2.5-3.0 b.y.) have been mapped along the basin edge by Boyce [1976] (see Figure 4, caption). Unit II lies stratigraphically between units I and III [Howard et al., 1973] and is assigned an age of about 3.5 b.y. [Meuhuberger, 1974]. On the basis of the stratigraphic and radiochronometric history, rille formation appears to have been limited to the period from about 3.5 to 3.7 b.y. ago, while downwarping and mare ridge formation continued to a later period.

Information on the approximate thicknesses of each unit comes from stratigraphic and remote sensing sources. The Apollo Lunar Sounder experiment revealed subsurface layering in several regions of the moon, including Mare Serenitatis. A trace along the southern edge of Mare Serenitatis (A-A' in Figure 4) revealed the presence of two subsurface reflectors at average depths of 900 m and 1600 m [May et al., 1976]. Phillips and Maxwell [1978] correlate the surface projection of the second reflector (1.6 km deep) with the edge of the dark mantle in the southeast (contact between units I and II) and possibly with the dark mantle deposit in the west. On the basis of this correlation, unit II has an average thickness of 0.7 km in the southern part of the basin. We adopt an average thickness of 1.0 km for the basin as a whole. The remaining mare fill occurs between the upper surface of the unfilled basin and the base of unit II. Photogeologic studies of Orientale [Greeley, 1976] show that early volcanic fill is characterized by flood basalt style eruptions in the central basin and sinuous rille-fed plains style eruptions along the rim. Using Orientale geometry as a guide (Figure 3), we adopt a multilayer approximation for the flooding of Serenitatis (Table 2).

Distribution of Tectonic Features. The shallow linear
trenches forming the linear rilles in this region are less than 2-3 km in width and are several kilometers to over 100 km in length. Their linear nature, the parallelism of their walls, and the similarity of floor and rim material all argue for a tectonic origin similar to terrestrial graben structures. Linear rilles on the moon form in both the maria and the highlands and are essentially restricted to the nearside, showing two modes of occurrence: (1) concentric to mare basins and (2) regional systems generally tangent to basins [Scott et al., 1975]. Although many rille systems are generally concentric to the maria in mare basins, in detail they often appear to follow preexisting structural grains rather than the exact edge of the mare fill [Head, 1974a]. In the vicinity of Serenitatis, linear rilles are concentrated in the southern and eastern parts of the basin (Figure 6) and are dominated by patterns concentric to Serenitatis [Head, 1974a]. In the southwest a series of rilles occurs in the dark mantle (Rima Sulpicius Gallus) and in unit I and must have extended further along the edge of the basin prior to the emplacement of unit II. These rilles extend along the mare edge from Menelaus to the dark mantle region near Apollo 17, where they branch. One set trends along the basin edge in mare and highlands to Posidonius crater, while the other extends in a broader arc into the adjacent uplands [Head, 1974a]. The Serenitatis rille systems define a belt about 50-100 km wide. Stratigraphic relationships indicate that the rilles were flooded by emplacement of unit II, but their full original extent around the basin interior is unknown. The linear rilles are believed to have formed in response to the downwarping of central Serenitatis [Howard et al., 1973; Head, 1974a; Muehlberger, 1974].

Mare arches (broad linear swells with smooth slopes) and mare ridges (narrow linear ridges with sharply defined edges) combine to form extensive ridge systems in Serenitatis (Figure 7). Although they occur predominantly in the mare, ridges can extend into adjacent uplands. Maxwell et al. [1975] have recently described the distribution and morphology of ridge systems in Serenitatis. The most prominent aspect is a roughly circular system of ridges approximately 400 km in diameter and generally concentric to the basin and its mare fill. The system is discontinuous and formed of segments arranged in various directions. Between this circular structure and the basin edge, numerous smaller ridge systems show a generally radial orientation, although abundant additional concentric segments also occur. Local circular arrays of ridge systems are believed to be due to underlying craters. Plots of ridge and arch segment orientations show, in addition to a generally even distribution, a distinctive north-south peak. On the basis of regional distribution and subsurface structure, Maxwell et al. [1975] attribute the formation of mare ridge systems to gravitational readjustments of mare fill along Serenitatis basin and prebasin structures (outer edge of thick mare fill, ancient crater structure, and regional fracture systems). A study of several specific ridges and scarp in Serenitatis [Lucchitta, 1976] yielded similar conclusions. Formation of ridge systems in Serenitatis spanned the time of emplacement of several mare units. They occur predominantly in unit III, less often in units I and II. The time of onset of ridge system formation is difficult to establish. Although discontinuities and changes in morphology occur along ridges at contacts between units, these may be due in part to thickness or material property differences. Ridge formation continued past the emplacement of unit III, since the surface is deformed extensively by ridge system development.

Summary. We propose the following volcanic and tectonic evolution for Serenitatis. Closely following the formation of the Serenitatis and Imbrium basins, extensive flooding of Serenitatis occurred 3.85-3.65 b.y. ago, filling the central basin to significant depths (unit I). Pyroclastic deposits (dark mantle) accompanied the emplacement of this unit. Subsidence of the newly emplaced lavas was accompanied by tilting of mare surfaces and formation of basin concentric grabens in unit I and surrounding uplands in the period 3.7-3.5 b.y. ago (Figure 6). Emplacement of unit II followed at about 3.5 b.y. and filled the centrally subsiding depression with approximately 1 km of additional lava. This unit extended out to the edge of the basin in several areas and flooded and embayed many of the graben developed in unit I. Subsidence continued after the emplacement of unit II, and unit III formed 3.4-3.0 b.y. ago by flooding of the innermost part of the basin to an average depth of about 1 km. Mare ridges are well developed in units II and III (Figure 7) and continued to form after the final emplacement of unit III ~3.0 b.y. ago owing to its continued downwarping. Emplacement of several patches of mare younger than unit III, mapped around the edge of Serenitatis by Boyce [1976], may also have occurred somewhat more recently than ~3.0 b.y. ago.

![Fig. 3. Impact basin topography. Shown are (a) Orientale cross section, from Head et al. [1975], and (b) cross section of an unfilled Serenitatis basin constructed on the basis of ridge assignments of Head [1978] and appropriate dimensional scaling. OR denotes Inner Rook; OR, Outer Rook; C, Cordilleria. In Serenitatis, ring 1 is mare ridge ring, ring 2 is Haemus Mountains (see Table 1, Figure 2b).](image)

![Fig. 4. Geologic map of Mare Serenitatis, adapted from Howard et al. [1973]. Major mare basalt units are listed in the general order of emplacement (from dark mantle unit I, oldest, to central basalts unit III, youngest). Additional data suggest that two smaller mare areas may postdate the central basalts: the subcircular area of unit II in northwest Serenitatis and a region of unit II along the annulus just south of Posidonius [Thompson et al., 1973; Boyce, 1976]. The line A-A' indicates the surface trace of the Apollo Lunar Sounder experiment.](image)
TABLE 2. Models of Loading of the Serenitatis Basin

<table>
<thead>
<tr>
<th>Unit I Time</th>
<th>Unit III Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial Distance x, km</td>
<td>Fill Thickness, y, km</td>
</tr>
<tr>
<td>0–115</td>
<td>6.5</td>
</tr>
<tr>
<td>115–140</td>
<td>4</td>
</tr>
<tr>
<td>140–200</td>
<td>2</td>
</tr>
<tr>
<td>200–240</td>
<td>1</td>
</tr>
<tr>
<td>240–295</td>
<td>1</td>
</tr>
</tbody>
</table>

LOCAL STRESS: MARE LOADING

The close association of linear rilles and of ridges with mare basins clearly points to a controlling source of stress that is local to the mare region. The demonstration that rille and ridge formation is linked to mare basin subsidence indicates that this local source of stress is loading of the lithosphere by mare basalt fill. Both the magnitude of the superisostatic volcanic load and the response of the lunar lithosphere to that load likely varied from mare to mare and within each mare as a function of time. That the oldest exposed mare basalt units record evidence for mare subsidence in their basin-facing topographic slopes and their concentric graben [Bryan, 1973; Muehlberger, 1974] attests to the importance of superisostatic crustal loading for at least short time intervals as early as 3.7–3.8 by. ago. The near-surface excess mass in the mascon maria [Phillips et al., 1972] indicates that superisostatic loading in these maria has persisted until the present [Wise and Yates, 1970; Wood et al., 1970]. The stress beneath mascon maria has been estimated by Conel and Holstrom [1968] and O'Keefe [1968] using elastic half-space models and by Kuckes [1977] using plate flexure theory.

The effect of mare basalt loading on stress near lunar maria is modeled in this paper by using the solution for stress in a spherical liquid-filled elastic shell due to a surface load with cylindrical symmetry [Brotchie and Silverst, 1969; Brotchie, 1971]. The details are given in Appendix 1. Such a model for the lunar mare basins must be regarded as idealized, for several reasons. The model is for an elastic shell of constant thickness and thus ignores potentially important variations in lithospheric thickness due, for instance, to the basin formation process. The interior of the shell is treated as an inviscid fluid, so that the viscous or viscoelastic response of the underlying material is neglected. Finally, the assumption of perfect elasticity ignores the effect of failure for which there is clear geologic evidence. Incorporation of a yield strength and an elastic-plastic rheology will generally lower the estimates of stress from those given here. Some of these points are discussed further in later sections.

The vertical displacement and horizontal surface stresses for a lunar lithosphere subject to a uniform cylindrical load are shown in Figure 8. The load is 200 km in radius and has a magnitude of 1.3 × 10^4 dynes/cm^2; these parameters provide a rough approximation to the excess mass associated with the Serenitatis mascon [Phillips et al., 1972]. This simple model is used only to illustrate the general form of solutions to surface loading problems; more detailed models of loading for the mare basalt fill in Serenitatis are given below. The displacement w and the surface horizontal radial and azimuthal stress components, σr and σθ, respectively, with respect to the load center are determined from (A3)–(A14). Necessary parameters are Young’s modulus E and Poisson’s ratio ν for the elastic shell, the density ρ interior to the shell, and the shell thickness T. We have taken ρ = 3.4 g/cm^3, ν = 0.25, and E = 10^8 dyne/cm^2, the last figure equivalent to the value determined seismically for the moon’s lower crust [Tokóz et al., 1974]. The thickness of the elastic lithosphere is treated as an unknown to be determined from observations. As on the earth [e.g., Turcotte, 1974], the thickness of the elastic portion of the lithosphere is expected to be less by a factor of perhaps 2–4 than the thickness of the lithosphere defined thermally or seismically or than the depth to partially molten zones.

The displacement w in Figure 8 is positive (downward) for an area larger than that of the load, and the region outside the zone of subsidence is slightly uplifted [Brotchie and Silvester, 1969]. Beneath the load center the horizontal stress components σr and σθ are equal and negative (compressive). At the edge of and outside the load, σr is large and positive (extensional), while σθ is generally smaller and negative. Beneath the load and near the load edge, σr and σθ are dominated by the contribution of the bending moments M and Mθ in (A13) and (A14). Far from the load, the major contribution to σr and σθ is the membrane stress described by the stress resultants Nθ and Nσ.

Figure 8 demonstrates the importance of the elastic lithospheric thickness T in controlling both the absolute levels of displacement and stress and their variation with distance from the load center. Both the radius at which σr is a maximum and the value of that maximum, for instance, are strong functions of T. Since the lithospheric thickness is not well known a priori for the moon as a function of space or time, the sensitivity of the stress field to T should allow that parameter to be estimated by a comparison of calculated stresses with the visible manifestations of the actual stress field at the lunar surface.

Near the load center, the fact that the components σr and σθ are both compressive and comparable in magnitude should lead, for sufficiently high stress levels, to compressive failure features either with no strongly preferred orientation or with an orientation controlled by preexisting zones of weakness or by a superposed regional stress field. At radial distances where σr and σθ are both compressive but σr is significantly larger in magnitude, compressive features with a radial orientation will be favored. At radial distances where σr is large and extensional the predicted failure mode will be normal or strike slip faulting, depending on whether the vertical or the azimuthal stress component is the greatest compressive stress. A wide annular zone of graben concentric to the basin will result from the presence of a modest regional stress field of horizontal extension.

With the results of simple load models as shown in Figure 8 as a guide, we may examine the consequences of models for the basin loading in Serenitatis that take into account both the prefill basin topography and the history of volcanic deposition. Two models are considered in detail here: one appropriate to the time between the emplacement of units I and II, a time when basin subsidence led to graben formation in the outer mare deposits and in the adjacent highlands, and one appropriate to the time just after the emplacement of unit III, a time when mare ridge formation was the only expression of continued basin movement.

The radial horizontal surface stress for a model of loading of the Serenitatis basin after unit I emplacement is shown in Figure 9 for several possible values of the elastic lithospheric thickness. The load model, given in Table 2, is based on the following assumptions: (1) the prefill basin topography for Serenitatis was as shown in Figure 3b; (2) the basalt after unit I...
emplacement not only filled the entire depression within the inner ring (to a thickness of ~5.5 km) but also contributed an additional layer of fill ~1 km thick within the second ring (Outer Rook Mountain analog); (3) the superisostatic load at the basin center, or that load in excess of the fill needed to balance the mass deficiency of the basin topography, was equal to the typical current load of comparable size mascon basins, 850 ± 50 kg/cm² or $1.4 \times 10^9$ dyne/cm² [Sjogren et al., 1972, 1974; Phillips et al., 1972; Sjogren and Wollenhaupt, 1973; Muller et al., 1974], and was contributed by all basalt layers in proportion to their thickness. The third assumption is quantitatively arbitrary and is purely for the convenience of showing stresses with actual magnitudes; it is equivalent to the supposition that about one third of the early basalt fill was superisostatic. The magnitudes of the stresses shown scale linearly with the assumed load; the relative change in stress with radial distance from the basin center (including the location of maxima and minima) does not change as long as the relative distribution of load with radial distance is fixed.

The main geologic constraint on a loading model for this time period is that the stresses be capable of forming concentric graben in the still exposed unit I deposits at the outer edge of Mare Serenitatis and in the adjacent highlands (Figure 6). The state of stress in the portion of the maria subsequently covered by units II and III is unconstrained. The region of large extensional stresses predicted by a model outside the basin, however, should not extend beyond the limit of observed basin concentric rilles in the highlands, about 400-km radial distance from the basin center. A further constraint is that the predicted subsidence due to superisostatic loading at unit I time be less than or equal to the total thickness of unit II (~1 km), under the assumption that the unit I and unit II surfaces were each approximately level at the time of their emplacement. This latter constraint is only a loose guide in view of the lack of specific information on the absolute magnitude of the load.

The models in Figure 9 with an elastic lithospheric thickness $T$ in the range 25–50 km, with values toward the lower end of
that range preferred, satisfy the observed location of graben and the loose constraint that the average basin subsidence be less than ∼1 km. A larger value for $T$ (e.g., $T = 100$ km) results in a broad zone of large and extensional $\sigma_2$ that extends to too great a radial distance from the basin center. The region of maximum $\sigma_2$ in the figure is characterized by small negative (compressional) values for $\sigma_2$ according to the simple flexure models, so that normal faulting and graben formation are the predicted failure modes only in the presence of mildly extensional regional stress (see below). The stress distributions shown in Figure 9 for $T = 25$–50 km lead to the prediction that additional graben were formed in the mare surface at unit I time closer to the basin center than those still exposed, but such features would have been buried by later basalt fill.

It should be repeated that the stresses shown in Figure 9 are hypothetical elastic stresses and will be lessened by lithospheric failure. The stress magnitudes exceed typical extensional strengths of rock, and the observed graben testify to the fact that failure did occur. Thus curves such as those in Figure 9 should be used only as a guide to the most likely location of failure sites rather than as an accurate indicator of instantaneous stress.

It is also worth noting that the stresses shown are for an elastic lithosphere of constant thickness. Because of the basin excavation and the frequent or continuous episodes of volcanism that filled the basin in ∼0.1 b.y., the lithosphere beneath Serenitatis at unit I time is likely to have been thinner than that beneath typical nonmare areas. To first order, a thinner lithosphere tends to result in more concentrated stresses and more accentuated bending moments. A gradual increase in lithospheric thickness from basin to adjacent highlands at unit I time may be inferred from the increase in spacing between adjacent rilles as one moves from the mare to the terrae (Figure 6) if fracture spacing in general varies directly as plate thickness [Vogt, 1974].

The radial horizontal surface stress for a model of the loading of the Serenitatis basin after unit III emplacement is shown in Figure 10, again for several possible values of $T$. The load model, given in Table 2, is based on the following assumptions: (1) unit II, ∼1 km thick, extends to the second basin

![Fig. 6. Major linear rilles developed in Mare Serenitatis and surrounding uplands.](image)

![Fig. 7. Major mare ridges developed in Mare Serenitatis.](image)
ring; (2) unit III, \( \sim 1 \text{ km} \) thick, covers the central portion of the mare to a radial distance of 240 km; (3) the superisostatic load at the basin center is the current mascon load, \( 1.3 \times 10^8 \text{ dynes/cm}^2 \) [Phillips et al., 1972], and is contributed by units II and III, assumed to have a density of 3.4 g/cm\(^3\), and by an appropriate contribution from the earlier fill in proportion to fill thickness. The last assumption is equivalent to the statement that units II and III are taken to be superisostatic and that most of the load adopted for unit I time is taken to have been relaxed by later basin subsidence. Assuming that the load at a time shortly after unit III emplacement is equal to the current load gives a conservative estimate because relaxation and partial compensation of the load in the last 3 b.y. have been ignored.

The primary geological constraint for this time period is the formation of mare ridges in the locations and with the orientations currently observed. A second constraint is that graben formation had ceased by this time, but we postpone discussion of this requirement until after consideration of the global stress (see the next section). A further constraint is that the total subsidence following unit III emplacement not exceed the observed departures of \( \sim 0.4 \text{ km} \) of the current mare surface from a level one.

Compressive values for \( \sigma_r \) and \( \sigma_\theta \) of a substantial magnitude (0.5 kbar) are predicted for most of the mare for an elastic lithospheric thickness \( T \) in the range 50–100 km. The area of predicted compressional failure would be expanded in the presence of a global compressional stress field. A value for \( T \) of 25 km or less does not give large enough or extensive enough compressive stresses and can be rejected for this time period. A value of \( T \) nearer 100 km than 50 km is preferable on the grounds that the predicted subsidence (\( \sim 0.5 \text{ km} \) versus \( \sim 1 \text{ km} \), respectively) is closer to that observed.

The difference in the best fitting values for \( T \) between the models of Figure 9 and those of Figure 10 is significant and likely reflects a rapid growth in effective lithospheric thickness between the time shortly after basin excavation and the time when mare volcanism was drawing to a close. Such a rapid growth is reasonable on thermal grounds [Arkani-Hamed, 1974]. A value of 50–100 km for \( T \) was also derived by Kuckes [1977] from the modeling of mascon gravity data in terms of plate flexure theory.

As was noted with the uniform cylindrical loading models, the near equivalence of \( \sigma_r \) and \( \sigma_\theta \) in the central portion of the mare leads to preferred orientations for compressive features such as mare ridges only if other factors are acting to accentuate the stress in particular locations or directions. The mare ridge ring, for instance, likely arises because of the concentration of stress in the relatively thin mare deposits above the inner peak ring. The preferred north to northwest orientation of ridges in central Serenitatis [Maxwell et al., 1975] may result from enhanced east-west compression, perhaps associated with tidal despinning [Worrall et al., 1978]. In the outer portions of the maria, however, \( \sigma_\theta \) is significantly more compressive than \( \sigma_r \), which favors radial orientation of compressive failure features, in general agreement with observed ridge orientations.

### Global Thermal Stress

To the local stress field near mare basins discussed in the last section must be added any stress on a regional or global scale. Though there are several possible sources of stress on such
scales, the primary source considered here is the thermal stress associated with the differential expansion and contraction of the lunar interior with time. The lithospheric stresses arising from particular thermal history models for the moon have been given by MacDonald [1960], Solomon and Chaiken [1976], and Solomon [1977]. The theory for thermal stress in a planet composed of two layers with differing elastic properties (e.g., the lunar crust and mantle) is given in Appendix 2.

The observation that the thermal stress in the moon never was large enough to produce global scale extensional or compressional failure of the lithosphere since cessation of heavy bombardment greatly restricts the possible thermal histories of the moon. Thermal evolution models that produce acceptably small stress and volume changes in the last 4 b.y. predict a period of modest expansion and surface horizontal extension followed by modest contraction and more compressive horizontal surface stress [Solomon and Chaiken, 1976; Solomon, 1977]. The time when the moon switched from an expansion stage to a contraction stage, the time of peak lunar volume, is constrained by the global stress limit only to lie within the broad and approximate time interval 1-3 b.y. after lunar formation.

For a subset of the thermal history models consistent with the absence of global scale tectonics the timing of rille and ridge formation in mare basin regions can be explained by the superposition of local and global stress fields, as is discussed in the next section. The thermal stress and lunar radius are shown as functions of time for one such model in Figure 11. The thermal history model starts from an initial temperature profile with melting to 300-km depth and a cool deep interior; the initial central temperature is 300°C. The model is otherwise similar to the suite of thermal models given in Solomon and Chaiken [1976] and Solomon [1977]. In particular the adopted heat source abundances (\(U = 40\) ppb present-day bulk concentration) satisfy the Apollo heat flow measurements [Langseth et al., 1976].

The period of volumetric expansion and horizontal extensional surface stress is confined to the first 1 b.y. of lunar history in the model. After 3.5-3.6 b.y. ago, the time of peak lunar volume, the cooling of the outer portion of the moon dominates the warming of the deep interior, and global contraction produces increasingly compressive horizontal stress at the lunar surface. The peak extensional and compressive stresses following cessation of heavy bombardment (\(\sim 200\) bars and \(\sim 1\) kbar, respectively) are marginally small enough to be consistent with the absence of moon-wide lithospheric failure. The timing of stress changes, however, is important in controlling the temporal relationships of tectonic features near mare basins, a topic which we can now explore in quantitative detail.

### Tectonic Synthesis

The stress field in the vicinity of mare basins is the sum of the local stresses due to mare basalt loading plus the global thermal stress and any other large-scale stresses. The global stress \(\sigma_t\) will add a constant to both \(\sigma_r\) and \(\sigma_\theta\), as is shown in Figures 9-10, and can result in a significant change in the magnitudes and possible changes in the signs of the horizontal stresses radial and azimuthal to mare basins. As is shown in Figure 9, the accumulated horizontal stress from 3.9 to 3.6 b.y. in the model of Figure 11, about 170 bars extension, must be added to the stresses shown to be able to predict the preferred locus and mode of failure. Such a stress level is comparable with or larger in magnitude than the small compressive values.
of $\sigma_r$ at 250- to 400-km radial distance from the Serenitatis center, so that normal faulting or graben formation rather than strike slip faulting is predicted for that distance range and time period. From 3.6 b.y. until the present in the model of Figure 11, $\sigma_r$ becomes more compressive. For instance, the accumulated horizontal stress from 3.6 to 2.4 b.y. according to the thermal model, about 620 bars compression, is shown in Figure 10 as a constant to be added to the stresses shown after unit IIII emplacement.

The superposition of local and global stresses provides a straightforward and quantitatively viable explanation for the timing of rille and ridge formation in the Serenitatis basin area (Figure 12). From the time when basin filling reached high enough levels to initiate subsidence until the time at or shortly after peak lunar volume the global extension acted to accentuate the extensional stress at the edge of the mare basin due to loading, and concentric graben formed as this stress was relieved. After peak lunar volume the global compression acted both to reduce and eventually suppress the zone of extensional stress outside the mare load and to enhance the horizontal compression within the load area. Thus for this later time period, formation of mare ridges without concurrent rille formation was the tectonic expression of continued basin subsidence due to renewed loads [c.f. Muehlberger, 1974].

Other major mare basins on the lunar nearside display a history of mare basalt emplacement, concentric graben formation, and mare ridge formation generally similar to that outlined in this paper for Serenitatis [Head and Solomon, 1979]. In particular, Lucchitta and Watkins [1978] have documented that lunar graben ceased to form at 3.6 \pm 0.2 b.y. ago moon wide, even though mare volcanism, basin subsidence, and mare ridge formation all continued to later times. The global synchronism for cessation of graben formation underscores the significance of the global stress field in controlling the timing of basin-related tectonic features.

It is important to emphasize that the formation of most, if not all, linear rilles and mare ridges is not simply a result of global-scale expansion or contraction. The superisostatic loading of mare basins and the consequent vertical subsidence as the lithosphere adjusted to the load are the primary causes of basin-related graben and of mare ridges. The global thermal stress is an important modulator of the local stress field, however, and can affect whether rilles or ridges are the more likely to form at a particular time.

The explanation for the spatial and temporal occurrence of rilles and ridges in Serenitatis and other basin areas provides new limits on possible thermal history models for the moon, limits even more stringent than those imposed by the absence of global-scale tectonic features. In particular, the time of peak lunar volume cannot occur substantially more recently than 3.6 \pm 0.2 b.y. ago in order to restrict lunar rille formation to before that time [Lucchitta and Watkins, 1978]. Models with peak volumes occurring somewhat earlier than the time shown for the model in Figures 11 and 12 are permitted, as long as the global thermal stress was not large and compressive in the time interval (~3.5-3.8 b.y.) during formation of most graben still preserved. Stated in another form, models with initial temperature profiles slightly warmer than the profile of the model in Figures 11-12 are still consistent with mare basin tectonic relationships. Substantially hotter initial models, however, are excluded because of their prediction of large compressive stresses on a global scale due to the cooling of the moon in the last 4 b.y.

**Some Further Ramifications**

The concepts of loading and subsidence of mare basins and of a local stress field superposed on a time varying global stress field lead naturally to the consideration of two related subjects, at least on a qualitative level. These are the relationships among mare basalt volume, basin load, and lithospheric response with time, and the locus and timing of mare volcanic eruptions.

The thickness of mare basalt versus time in central Mare Serenitatis is shown schematically in Figure 13, based on the expected prefll basin topography (Figure 3b), the surface exposure and relative ages of the major basalt units (Figure 4), and the correlation of subsurface radar reflectors with mapped basalt units [May et al., 1976; Phillips and Maxwell, 1978]. The time intervals over which units I, II, and IIII were erupted is not known; the actual curve for basalt thickness versus time may be more or less stepped than that shown.

At some time before the end of unit I emplacement the basalt fill constituted a superisostatic fill, and the basin began to subside (Figure 13). The load was continually modified both by the lithospheric response and by the addition of new fill. In abstract terms, suppose that a single unit load is applied at a time $\tau$ and that $g(t - \tau, \tau) \leq 1$ is the load at time $\tau$. The response function $g$ depends upon $\tau$ through the viscosity of the lunar asthenosphere and through inelastic deformation of the lunar lithosphere. The function $g$ also depends upon $\tau$ because the rheology of the moon in general is changing in response to cooling of the crust and upper mantle. One expression of this change, for instance, is the increase in the best fitting elastic lithospheric thickness with time in the Serenitatis model of Figures 9 and 10. The load on a mare basin in which a fill load $\Delta q(\tau)$ has been added will be $\int \Delta q(\tau) g(t - \tau, \tau)d\tau$. A similar expression will define the vertical subsidence. These relations are shown schematically in Figure 13.

After emplacement of the youngest major mare unit, sub-

![Figure 12](image_url)
addition to controlling the tectonics, may also exert a strong influence on the eruption sites of mare basalt magma. The case may reasonably be made that magma ascent is more likely in the presence of horizontal extensional stress than in the presence of horizontal compression. This statement provides a link between periods of planetary expansion and episodes of widespread volcanism on the terrestrial planets [Solomon, 1978]. The concept should also be valid in a local setting.

Shown in Figure 14 is a schematic path of magma ascent beneath a basin loaded superisostatically by mare basalt fill. At the base of the lithosphere, magma will tend to rise easiest beneath the load center, where the bending stresses are most extensional. In the upper half of the lithosphere the magma ascent will be most favorable near the load edge. A prediction of the scenario depicted in Figure 14 is that the most likely eruption site in the later stages of mare fill is near the edge of the mare basin. This prediction is in agreement with the mapped eruption sites of distinct mare basalt flows in Imbrium [Schaber, 1973; Schaber et al., 1976] and with the observations of small regions of relatively young mare deposits at the outer edges of several maria, including Serenitatis [Boye, 1976; Boyce and Johnson, 1977, 1978]. The reduction in the extensional bending stress due both to growth of the lithosphere and to the superposition of an ever more compressive global horizontal stress will serve eventually to shut off mare volcanism entirely.

CONCLUSIONS

The major mare basins on the moon have a well-documented history of repeated vertical movement and subsidence-related tectonic features in response to superisostatic loading by mare basalt fill. Linear rilles concentric to the basins and mare ridges within the basalt units are both products of this excess load. The restricted time interval for rille formation, in contrast to the generally later and probably longer time period for ridge formation, is testimony to the importance of a time-dependent global stress field in controlling the tectonic response to basin subsidence.

Quantitative models for stress in the Serenitatis basin area, including the global thermal stress associated with lunar thermal history, are presented in this paper and can explain the spatial distribution, the orientations, and the formation times of rilles and ridges in that region. Such an explanation allows important new constraints to be placed on the isostatic response function of the lunar lithosphere, in particular the time dependence of the elastic lithospheric thickness beneath Serenitatis. These models also provide new and stringent limits on possible lunar thermal histories because of the requirement that peak lunar volume occur at or before 3.6 ± 0.2 b.y. ago. Finally, the models yield a simple explanation for the local-
ization of the most recent eruption sites of mare basalt magmas to mare basin edges.

The concept of mare loading and superposed global stress can be applied to other major mare basins on the moon and provides an explanation of tectonic features associated with those regions and further evidence of spatial and temporal variations in lunar lithospheric thickness [Head and Solomon, 1979]. The same concept can also be applied to major basins on other planets, where both the global thermal history and the basin-related tectonic history are different from those on the moon.

APPENDIX A: LITHOSPHERIC STRESS DUE TO SURFACE LOADING

Consider a thin, spherical, elastic shell of thickness $T$, Young’s modulus $E$, and Poisson’s ratio $\nu$ overlying a fluid interior of density $\rho$. Let $R$ be the spherical radius to the midplane of the shell, let $g$ be the gravitational acceleration at radius $r = R - T/2$, and let $D$ be the flexural rigidity of the shell

$$D = \frac{ET}{12(1 - \nu^2)}$$

(A1)

Consider a uniform vertical load $q$ applied to a circular area of radius $F \ll R$ on the surface of the shell. The solution for vertical displacement and stress has been given by Brotchie [1971], with some typographical errors, in terms of the polar coordinates radius $R$ and azimuth $\theta$ in the horizontal plane with an origin at the load center. The polar radius is expressed as the nondimensional quantity $x$ by normalizing by the ‘radius of relative stiffness’

$$l = \left(\frac{D}{ET/R^4 + \rho g}\right)^{1/4}$$

(A2)

Inside the load ($x \leq d = F/l$), the vertical displacement $w$, the radial and azimuthal bending moments $M_r$ and $M_\theta$, and the radial and azimuthal stress resultants $N_r$ and $N_\theta$ are given by

$$w = \frac{q}{(ET/R^4 + \rho g)}(d \text{ Ker} - d \text{ Kei' d Kei} + 1)$$

(A3)

$$M_r = -qd\left[\text{Ker} \left(\text{Ber} x + d \text{ Kei} \right)ight]$$

(A4)

$$M_\theta = -qd\left[\text{Ker} \left(\nu \text{ Ber} x + d \text{ Kei' Ber} x\right)\right]$$

(A5)

$$N_r = \frac{qET/R}{ET/R^4 + \rho g} \left[\frac{1}{2} + \frac{d}{x^2} \left(\text{Ker} \text{ d Kei' x + Kei'} \text{ Ber} x\right)\right]$$

(A6)

$$N_\theta = -N_r + (ETw/R)$$

(A7)

where Ber, Bei, Ker, and Kei are Bessel-Kelvin functions of order zero [Abramowitz and Stegun, 1964] and the prime denotes the first derivative. In (A3), $w$ is positive if it is downward (inward).

Outside the load ($x \geq d$), the displacement, moments, and resultants are given by

$$w = \frac{qd}{(ET/R^4 + \rho g)}(\text{Ber} \text{ d Ker} x - \text{Ber} \text{ d Kei} x)$$

(A8)

$$M_r = -qd\left[\text{Ber} \left(\text{Ker} x + \frac{1 - \nu}{x} \text{ Ker} x\right)\right]$$

(A9)

$$M_\theta = -qd\left[\text{Ber} \left(\nu \text{ Ker} x + \frac{1 - \nu}{x} \text{ Kei' Ker} x\right)\right]$$

(A10)

$$N_r = \frac{qET/R}{ET/R^4 + \rho g} \left[\frac{1}{2} + \frac{d}{x^2} \left(\text{Ber} \text{ d Ker} x + \text{Ber} \text{ d Ker} x\right)\right]$$

(A11)

$$N_\theta = -N_r + (ETw/R)$$

(A12)

Within the shell at $(R - T/2) \leq r \leq (R + T/2)$ the net horizontal stresses radial and azimuthal with respect to the load center are given by

$$\sigma_r = \frac{N_r}{T} + \frac{12M_r}{T^3}(r - R)$$

(A13)

$$\sigma_\theta = \frac{N_\theta}{T} + \frac{12M_\theta}{T^3}(r - R)$$

(A14)

respectively, where the stresses are positive in extension and negative in compression.

The displacements and stresses in a shell subjected to an arbitrary load with cylindrical symmetry about the load center may be determined by the superposition of (A3)-(A14) for a set of uniform cylindrical loads of variable $q$ and $d$ that approximate the actual load.

APPENDIX B: THERMAL STRESS

We solve for the thermal stress in a spherical elastic body consisting of two layers of differing elastic properties and a coefficient of thermal expansion that varies with radius $r$. Denote the outer shell, from $r = a$ to $r = b$, as layer 1 and the inner sphere as layer 2. Let $\Delta T(r)$ and $\alpha(r)$ be the temperature change and volumetric thermal expansion coefficient, respectively. In each layer the solution for displacement $u$, radial stress $\sigma_r$, and tangential stress $\sigma_\theta$ is of the form [Timoshenko and Goodier, 1970]

$$u(r) = \frac{2E}{1 + \nu} I(r) + \frac{EA}{1 - 2\nu} \frac{1 - \nu}{(1 + \nu)r^2}$$

(B1)

$$\sigma_r(r) = \frac{E}{1 + \nu} I(r) + \frac{EA}{1 - 2\nu} + \frac{EB}{(1 + \nu)^2} - \frac{E\alpha(r)\Delta T(r)}{3(1 - \nu)}$$

(B2)

$$\sigma_\theta(r) = \frac{E}{1 + \nu} I(r) + \frac{EA}{1 - 2\nu} + \frac{EB}{(1 + \nu)^2} - \frac{E\alpha(r)\Delta T(r)}{3(1 - \nu)}$$

(B3)

where $E$ and $\nu$ are Young’s modulus and Poisson’s ratio, respectively, where

$$I(r) = \frac{1}{1 - \nu} \frac{1}{r^2} \int_0^r \alpha(r') I(r) dr$$

(B4)

and where $\nu$ is taken as constant throughout the body.

The four constants of integration, $A_1$, $B_1$, $A_2$, and $B_2$ (where the subscript denotes the layer), are determined from the four boundary conditions: (1) $u = 0$ as $r \to 0$; (2) $\sigma_r = 0$ at the surface $r = b$; (3) $u$ is continuous at $r = a$; and (4) $\sigma_r$ is
continuous at $r = a$. These boundary conditions give the results

$$A_1 = \left( \frac{2E_0 - E_1}{1 + \nu} f(a) + \left( \frac{2E_1 + E_0}{1 - 2\nu} \right) f(b) \right) \frac{b^p}{a^q}$$

\(\pm \left( \frac{E_0 - E_1}{1 - 2\nu} + \left( \frac{2E_1 + E_0}{1 + \nu} \right) \frac{b^p}{a^q} \right) f(b) \frac{b^p}{a^q}
$$

(B5)

$$A_2 = \left( 1 + \frac{b^p}{a^q} \right) A_1 - f(b) \frac{b^p}{a^q}$$

(B6)

$$B_1 = sA_1 - f(b)$$

(B7)

$$B_2 = 0$$

(B8)

where

$$s = \frac{1}{2} \left( 1 + \frac{b^p}{a^q} \right)$$

(B9)

In practice, the temperature change $\Delta T(r)$ for some time interval $\Delta t$ is taken from a thermal history model, and the displacements and stresses in (B1)-(B3) represent changes in those quantities for the same time period.

For the moon, Young's modulus is taken from the finite strain solution for the isothermal bulk modulus $K$ and a constant $\nu = 0.25$. The finite strain and thermal expansion parameters for the crust and mantle are those tabulated in the work of Solomon [1977]; the pressure distribution is that for a 70-km thick crust having an STP density of 2.8 g/cm$^3$ and a mantle STP density so as to satisfy the total mass and radius of the moon for the present-day temperature profile predicted by the thermal history model in question. The thermal expansion coefficient in (B3) and (B4) is corrected for pressure $P$ by the expression of Birch [1952] from first-order finite strain theory:

$$\alpha/a_0 = 1 + \frac{P}{K} \left( \frac{\delta K}{aK} \right)$$

(B10)

where the subscript 0 denotes zero pressure.

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