
Introduction: Explosive volcanic eruptions are predicted to have occurred on Mars during its geologic history. We focus attention on the formation of a distinctive type of explosive volcanic product, accretionary lapilli, whose recognition in the geologic record during detailed surface exploration could provide important information about the nature of the volcanic history of Mars. Most treatments of the formation of fall deposits from volcanic eruption clouds on Mars have assumed that individual clasts were transported through, and fell from, such clouds as discrete objects with negligible mutual interactions. However, it is well established that the formation of accretionary lapilli, a mechanism that allows small particles to fall much more rapidly as members of clusters than as individuals, is an important factor in determining the spatial thickness and grain-size variations in fall deposits on Earth. We therefore explore the nature of this process in the martian environment.

Gilbert and Lane [1] showed that accretionary lapilli form in explosive eruption clouds when initially small particles grow by becoming coated with water condensing in the eruption cloud and accreting other small particles which they overtake as they fall. The overtaken particles become trapped in the water layer, and chemical processes bond the accreting particles to one another and ensure the stability of the growing structure. The critical factors controlling the growth of a lapillus to a final size \( D \) (assumed to be much greater than its initial size) are the vertical distance through which it falls, \( Z \), and the mass loading in the eruption cloud, i.e. the total mass of pyroclasts per unit volume, \( w \). Other important factors are the density of individual ash particles, \( \rho_p \), the porosity of the aggregates, \( \phi \), the density of the liquid or gas in the pore spaces, \( \rho_w \) and the aggregation coefficient, \( E \), combining the efficiency of sticking and the probability of collision, which accounts for the extent to which a small particle attempts to follow streamlines in the gas and be swept around any larger clast that overtakes it. The relationship is \( D = (0.5 w E Z) / \rho_w (1) \), where the lapillus bulk density \( \rho_b \) is equal to \( [\rho_p - \phi (\rho_p - \rho_w)] \). Two conditions are implicit in the assumptions behind this model. The first is that large pyroclasts fall through an eruption cloud faster than smaller ones. This will be true as long as the atmospheric pressure is large enough to ensure that particles interact with the gas according to the gas laws, falling with terminal velocities that are a function of their size. A comparison of the mean free paths of molecules in the martian atmosphere with the ranges of lapilli sizes considered here shows that it is only at heights greater than \( \sim 50 \) km on Mars that mean free paths exceed particle sizes so that the Knudsen regime becomes operative and all particles move at the same speed. This is not a limitation, because we assume that martian eruption clouds rarely rise to heights much greater than \( \sim 20 \) km. More important is the fact that a liquid water film is required on the colliding clasts to aid adhesion. This is common over a wide range of heights in eruption clouds on Earth, both where clasts are rising above the vent and where they are falling at greater lateral distances. However, it is more problematic on Mars because of the low (everywhere \( <240 \) K; [2]) atmospheric temperatures. Once a significant amount of atmosphere has been entrained into an eruption cloud on Mars any water is likely to be frozen to ice. However, we show that temperatures will be above the freezing point in much of the rising part of an eruption cloud, and we infer that it is here that most lapilli formation occurs. The fact that the relative motion of large and small clasts is superimposed on the high-speed turbulent motion of the rising part of the eruption cloud, rather than on the lower-speed, more nearly laminar motion of the atmosphere outside the rising core of the cloud, as on Earth, does not hinder the formation process and, indeed, may enhance it by providing an effectively greater path length over which particle collisions can occur.

Experimental data [1] show that \( E \) in equation (1) is \( \sim 1 \) for very small adhering particles and decreases from \( \sim 1 \) to \( \sim 0.1 \) as the adhering particle size increases from \( \sim 10 \) to \( \sim 100 \) \( \mu m \). \( \rho_p \) is taken as \( 1500 \) kg m\(^{-3} \) on Earth and Mars, and is \( \sim 50\% \) greater than \( \rho_w \) when water fills pore spaces and is very much greater than \( \rho_w \) when gas fills them. As a compromise we assume that pore spaces are half filled with water. \( \phi \) is generally \( \sim 0.4 \) for lapilli on Earth and there is no reason to expect values to be greatly different on Mars, so that \( [\rho_p - \phi (\rho_p - \rho_w)] = 1100 \) kg m\(^{-3} \). Application of eruption cloud models developed for the Earth to Mars suggests that such clouds should rise much higher in the atmosphere of Mars for a given mass eruption rate [3-5]. However, Glaze [6] pointed out that some of the assumptions made in these models about the mechanism of entrainment of atmospheric gases are not justified above \( \sim 20 \) km in the atmosphere of Mars, so that high discharge-rate eruptions generating clouds that might have been expected to convect to much greater heights may instead produce structures more analogous to the umbrella-shaped plumes on Io, where gas-particle interactions are minimal except near the surface. We therefore adopt 20 km as the maximum likely value of \( Z \) on Mars. Well above the vent in eruption clouds on Earth, \( w \) ranges from \( \sim 2 \times 10^{-3} \) to \( 10 \times 10^{-3} \) kg m\(^{-3} \). The major difference between explosive eruptions on Mars and Earth is that the volatiles exolved from the magma must eventually decompress by a much greater factor to reach equilibrium with the atmosphere on Mars than on Earth. Enhanced gas expansion implies that a given mass, and hence a given number, of pyroclasts will occupy a greater volume on Mars than Earth, leading to a smaller value of \( w \), which we now estimate.

It is likely that the vast majority of eruptions on Mars take place under conditions where the vent cannot flare outward toward the surface sufficiently rapidly to allow the pressure in the erupting jet of volcanic gas and entrained pyroclasts to decrease to the atmospheric pressure. Instead the eruption is choked, with the pressure in the vent being the value at which the velocity of the gas-pyroclast mixture is equal to the speed of sound in that mixture. Above the vent the pressure decreases through a complex series of shocks to reach the ambient atmospheric value; there is some lateral expansion of the jet, and the upward velocity of the gas and clasts increases before beginning to decrease again as the upward momentum of the erupted mixture is shared with entrained atmospheric gases.
The speed of sound, $U_s$, in the gas-pyroclast mixture is given by $U_s^2 = \left[ n (Q ) / m \right] \left[ 1 + [(1 - n) m P_f] / (n Q T \rho_m) \right]^2$ (2), where $n$ is the exsolved mass fraction of the major volatile in the magma (here assumed to be water vapor with molecular weight $m$ equal to 18.02 kg/kmol), $Q$ is the universal gas constant, 8314 J kmol$^{-1}$ K$^{-1}$, $T$ is the temperature of the eruption products, taken as 1450 K for a mafic magma, $\rho_m$ is the density of the magmatic liquid, taken as 2700 kg m$^{-3}$, and $P_f$ is the pressure in the choked flow in the vent.

The speed of sound is determined by the amount of expansion of the magmatic gas between the level at which the magma disrupts into pyroclasts and the vent. Using the common assumption that little gas exsolution occurs between the fragmentation level and the surface, and that fragmentation takes place when the volume fraction of gas bubbles in the magma exceeds a critical value of order 0.75, we find that the pressure at the fragmentation level is $P_l$ where $P_l = \left[ n (Q T) / \left[ 3 (1 - n) m \right] \right] \ln \left( (P_f / P_l) \right)$ (3). Equating the energy obtained from the decomposition of the eruption mixture between the pressures $P_l$ and $P_f$, to the kinetic energy of the eruption products gives [7] $U_s^2 = \left[ n (Q T) / m \right] \ln \left( (P_f / P_l) \right)$ (4).

The choked condition requires equating $U_s$ to $U_c$ at the vent, so that $\left[ 1 + [(1 - n) m P_f] / (n Q T \rho_m) \right]^2 = 2 \ln \left( (P_f / P_s) \right)$ (5). Given any choice of $n$ and hence $P_s$, this equation can be solved recursively by inserting an initial estimate of the value of $P_s$ (one half of $P_l$ is appropriate) into the right-hand side and solving the equation to obtain an improved estimate of $P_s$. After an adequate level of convergence has been obtained, either of equations (2) or (4) can be used to obtain the mean eruption speed $U_s$.

Above the vent, a series of shocks and expansion waves within the volcanic jet allows the pressure in the vent, $P_v$, to relax to the atmospheric pressure, $P_a$, over a vertical distance of at least several vent diameters. This process has been explored theoretically for expansions into a vacuum or into a crater-like structure around the vent, but is not well-studied for other geometries, especially where eruptions take place into an atmosphere which must eventually begin to be entrained into the jet. Nevertheless, a reasonable approximation to the amount of energy available from the decomposition from $P_v$ to $P_a$ is $\Delta E$ given by $\Delta E = \left[ (T / T_e) - 1 \right] \left[ n (Q T) / m \right] \left[ 1 - (P_f / P_v) \right]^{(3/2)}$ (6), where it has been assumed that the expansion and cooling of the gas-pyroclast mixture from its eruption temperature $T_e$ can be treated as the adiabatic expansion of a pseudogas having a ratio of specific heats $\gamma$ given by $\gamma = \left( s_{sp} + \lambda s_v \right) / \left( s_{sv} + \lambda s_v \right)$ (7) where $s_{sp}$ and $s_{sv}$ are the specific heats at constant pressure and constant volume, $\sim$3900 and $\sim$2800 J kg$^{-1}$ K$^{-1}$, respectively, of steam, $s_v$ is the specific heat at constant volume of the silicate material, $\sim$1000 J kg$^{-1}$ K$^{-1}$, and $\lambda$ is defined as $\lambda = (1 - n) / n$ (8). The energy $\Delta E$ must be added to the kinetic energy of the eruption products at the vent level to obtain their final velocity, $U_w$, after reaching atmospheric pressure from 0.5 $U_s^2 = \Delta E + 0.5 \Delta U_s^2$ (9).

Equations (2) to (5) involve only the magma water content, the magma temperature, and the physical properties of the water vapor, and so for any given magma, the values of $P_s$, $P_v$, and $U_c$ will be the same for choked eruptions on both Mars and Earth. However, the atmosphere pressure is involved in calculating the final velocity via equations (6) to (9) and so it is here that a significant difference appears between eruptions on Mars and Earth.

The value of $w$ is determined by the amount of expansion of the volatile phase, in this case water vapor, in the decompression region above the vent. The ratio $R$ of the volatile volumes before and after adiabatic expansion is given by $R = \left( P_s / P_f \right)^{(3/2)}$ (10) and is thus different on Mars and Earth, again because of the differing atmospheric pressures. The decompression is partly accommodated by the increase in gas velocity from $U_c$ to $U_s$ and partly by the increase of the cross-sectional area of the erupting jet, and it is this area change that is directly reflected by $w$. The factor $A$ by which the area increases, and hence $w$ decreases, is given by $A = R / \left( U_s / U_c \right)$ (11). The factor by which $w$ is smaller on Mars than on Earth is equal to $A_{M} / A_{E}$, and since the largest value of $w$ for terrestrial eruption clouds is $\sim$10 x 10$^{-3}$ kg m$^{-2}$, we divide this value by $A(w/A_E)$ to obtain the values of $w_M$ for Mars for each value of $n$. Finally, inserting these values of $w_M$ into equation (1), together with the values of the other parameters discussed earlier as being relevant to Mars, gives the values of the largest likely lapilli sizes, $D$. Values vary between -0.7 and 0.9 mm. The smallest lapilli sizes are likely to be smaller than these values by at least a factor of 10, and we note that all of the lapilli sizes are about one order of magnitude smaller than those commonly found on Earth.

The dispersal of accretionary lapilli with sizes that grow from $\sim$0.1 to $\sim$1 mm while falling through the martian atmosphere from heights of $\sim$20 km will depend on the wind regimes that they encounter while falling and on their terminal velocities. Wind speeds (dominated by zonal winds) are given for northern and southern summer conditions and for low and high dust loading [2], and typical conditions can be represented adequately by a wind profile that decreases roughly linearly from $\sim$40 m s$^{-1}$ at 20 km to zero at ground level. Clast terminal velocities depend on clast size ($d$, the diameter of an equivalent sphere) and bulk density $\rho_B$, on the density $\rho_s$ and viscosity $\eta$ of the atmosphere, both functions of height. Using the model in [2] we find the pressure, temperature, density and viscosity values, and the terminal fall velocities $U_f$ for several sizes of particles with density 1100 kg m$^{-3}$. These terminal velocities are calculated for the relevant regime (laminar or turbulent) in which the particles fall by taking $U_f$ to be the smaller of $U_f = (d^2 \rho_B g) / (18 \eta)$ (12), $U_f = (4 d \rho_B g / \left( 3 C_D \rho_s \right))^1/2$ (13), where $g$ is the acceleration due to gravity, $\sim$3.72 m s$^{-2}$, and $C_D$ is a dimensionless drag coefficient that depends on the shapes of particles but is of order unity. Using these values, incremental fall times between successive heights in the atmosphere are calculated and the windspeed $W$ is used to find the lateral displacement while traversing each height increment. These distances are then summed to give the lateral displacements while falling from 20 km. It is clear that the extent of dispersal of an accretionary lapilli fall deposit from an eruption cloud on Mars could range from several tens to many hundreds of km depending on the final particle size.