INITIAL RESULTS FROM USING CUBIC AND VARIATIONAL IMPLICIT INTERPOLATION FOR COMBINING HRSC STEREO TOPOGRAPHY WITH MOLA LASER ALTIMETRY. Prabhat\textsuperscript{1} and C. I. Fassett\textsuperscript{2}, \textsuperscript{1}Center for Computation and Visualization (prabhat@cs.brown.edu), \textsuperscript{2}Department of Geological Sciences (Caleb_Fassett@Brown.Edu), Brown University, Providence RI 02912, USA.

**Introduction:** Combining MOLA and HRSC topography information is useful because the two datasets give us knowledge about the Martian surface in very different ways. MOLA was a high-precision instrument but returned sparse data, and HRSC is somewhat less-precise but allows for creation of complete topographic models. In this work, we present some preliminary results from using simple interpolation functions (cubic and variational implicit) to combine the two sources of information. Our preliminary results look promising and warrant further investigation.

**Data sources:** MOLA: The MOLA instrument was a laser-ranging device; it operated by sending very short pulses of infrared radiation to the surface and measuring the time it took before it was reflected back to the spacecraft [1]. Before the laser ceased operation, MOLA made \textasciitilde660 million individual measurements of the Martian surface. Shots are separated by roughly 300 m along orbit, the footprint of MOLA shots is \textasciitilde100 m, and the geolocation of shots is known to better than 100 m. The vertical precision of MOLA is on the order of \textasciitilde0.5 m, and analysis of crossover shots of multiple orbits suggests the repeatability or drift in measurements is \textasciitilde\textasciitilde1 m [2]. MOLA-derived DTMs are derived by gridding of MOLA shot data, but obviously such grids are interpolated across regions where MOLA did not make measurements. Although the high-precision measurements made by MOLA have given us valuable information about Martian topography, the sparse coverage presents challenges, particularly in the more equatorial latitudes.

HRSC: The HRSC instrument has nine data channels for nadir imaging, photometry, color (red/green/blue/IR), and stereo imagery [3]. The nadir channel has resolution up to \textasciitilde12 m/pixel over a large, continuous image swath, which is ideal for interpreting surface geology. Using the HRSC stereo channels, it is possible to derive a scene-wide topographic model with spatial resolution of \textasciitilde50-200 m/pixel, depending on the observing conditions [3]. Unfortunately, the orbit of the Mars Express mission is not circular, so that the resolution of HRSC images and the derived DTM changes dramatically depending on the distance from the Martian surface. Derivation of stereo models requires knowledge of the camera optics (which are assumed stable) and tie points between image locations on the two stereo images. A surface topographic model can then be derived using the parallax between observations. An automated processing routine has been developed for HRSC data [7], but in practice the derivation of stereo models is computationally intensive and difficult. DLR produced “standard” 200-m resolution DTMs have been produced and released to the HRSC team, which we use as the reference HRSC model here.

There is a misfit of the HRSC DTMs to the MOLA points of tens to thousands of meters. There is thus a need to incorporate MOLA data into the HRSC processing to improve the DTM product. Recent work by Spiegel et al. [4] has begun to consider this problem, but their solution differs substantially from ours, in that they attempt to adjust the HRSC DTM to incorporate information derived from the MOLA gridded data. As outlined below, we attempt to combine the HRSC DTM with MOLA data using a constraint imposed by the MOLA shots themselves.

**Our approach:** Assuming that the input MOLA and HRSC data is perfectly registered, we would like to displace the HRSC grid, so that the ideal surface satisfies the following two properties:

1) it should pass through all the MOLA points

2) it should introduce minimal distortion to the dense HRSC grid.

We first calculate a displacement grid by taking the difference between MOLA points and the corresponding HRSC points. Since this grid is sparse, we can rephrase the goal as trying to fit a smooth surface to the displacement grid.

We have tried two approaches to solve this problem. We use MATLAB’s griddata function [5] with cubic interpolation and a variational implicit function [6]. For the results reported in this work, we use a basis function of the form \( f(x) = x^{3/2} \). The cubic interpolation functions maintain \( C^0, C^1 \) and \( C^2 \) continuity. The implicit function passes through the specified MOLA points while minimizing the aggregate squared curvature. While this is a desirable feature, this does not completely address the second property. Formulating an optimization function that would attain a compromise between both properties needs to be explored further in future work.

We tested our technique on two subsets of HRSC DTM data from the Arsia Graben region (Fig 3) and Nili Fosae region (Fig 1,2). The Arsia Graben dataset has 7324 MOLA points and 501x258 HRSC grid. The
Nili Fosae dataset has 8216 MOLA points and 298x241 HRSC grid. A summary of the quantitative results obtained on a Linux machine (2GHz, 4GB RAM) is presented below.

<table>
<thead>
<tr>
<th></th>
<th>Cubic interpolation</th>
<th>Implicit function</th>
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<tr>
<td>Time (s)</td>
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<td>Time (s)</td>
</tr>
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</tr>
<tr>
<td>Nili</td>
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References:


Fig. 1: Results for the Nili Fosae region. The raw MOLA data (left) and HRSC grid (center) is shown. Resulting mesh from variational implicit function is shown on the right. Note that the MOLA mesh on the left is generated using cubic interpolation for visualization purposes alone.

Fig. 2: Central figure shows cubic interpolation results. Figure on left shows the displacement applied to the HRSC grid under variational implicit interpolation; the figure on the right is the difference between the two methods.

Fig. 3: Results from Arsia Graben region. MOLA (left), HRSC(center) and implicit interpolation (right) meshes.