SPACE TIME LOCALIZATION OF UNDISCOVERED PLANETARY SATELLITES. N.I.Perov[1] and A.A.Nahodneva[2], [1]State Pedagogical University, Astronomical Observatory, Respublikanskaya, 108. 150000, Yaroslavl, Russia, e-mail: perov@yspu.yar.ru, [2]State Pedagogical University, Respublikanskaya, 108. 150000, Yaroslavl, Russia.

Introduction: Dynamical evolution of the Solar planetary system [2], [6], [3], [4], and since 1995 evolution of the exoplanets [9], is one of the main problems of celestial mechanics. Discovery, based on the observations, of extrasolar planetary systems (at May 2005 there had been found 150 planets like Jupiter) was one of the important results of the astronomy of the end of XX century and stimulated development of astronomy. The general direction of theoretical works, devoted to extrasolar planets, is improving of the cosmogonical models. The modern cosmogonists consider the discoveries of planets Earth-type and *minor bodies*, formed new planetary systems, will be in observing astronomy in the first and the second decades of the XXI century [7], [8].

We determine of regular, irregular, coorbital satellites of planets as natural celestial bodies, diameters of which are no less 1 km, and which are revolving around the planets (distance between the planet and the satellite is smaller by a factor of several orders in comparison with the distance between the planet and the Sun). In this case the «planet centric» force predominates over forces, are due the influence of the Sun, others planets and secondary satellites, oblateness of the planet, though the letters may set up significant perturbations of satellites orbits. It should be noted for the solar planets satellites the perturbations from the others planets are small, in comparison with perturbations of the Sun, and so others planets perturbations do not determine the motion of the satellites [2], [6].

In 1999-2005 many new small satellites of Jupiter (51), Saturn (28), Uranus (13) and Neptune (6) has been detected [4], [10], [11]. Opening this unusual collection of the satellites, discovered for the short interval of time, is an outcome of application of special methods for searching these objects and using of modern equipments, including 8.3 m telescope "SUBARU", permitting to scan across the great regions of sky near the giant planets [10].

In accordance with aforesaid it is very important and interesting to estimate by *theoretical way* numbers of unknown satellites of Saturn, Uranus, Neptune (the number of known satellites of Neptune remains almost constant for the decade and a half, after the flight of "Voyager-2" near this planet [5]) and satellites of the exosolar planets basing on the known parameters of these planets.

Oblateness of the solar planets and number of theirs satellites: In the table 1, made up with due account of observed data [4], [10], [11] connection between number (N) of the secondary satellites and geometric oblateness (α) of the Solar system planet is set up. N_o is known number of the observed planetary satellites and N_t (α) is a number of satellites, calculated by a formula (1)

$$N_t(\alpha) = -0.0213 + 312.4235\cdot\alpha + 10836.650\cdot\alpha^2 \quad (1)$$

Table 1. Oblateness (α) of the Solar system planets and number (N) of the planetary satellites (19.08. 05).

Planet	Oblateness, α	Number of observed satellites, N_o	Theoretical number of satellites, N_t (α)
Mercury	0	0	0
Venus	0	0	0
Earth	0,0034	1	1
Mars	0,0052	2	2
Jupiter	0,062	63+rings	63
Saturn	0,103	47+ rings	147
Uranus	0,06	28+ rings	58
Neptune	0,02	13+ rings	13
Pluto	?	1	?

The equation (1) is derived with help of the least square method (the planets and the satellites up Jupiter including are considered). The great number of the significant digits underscores the negligible influence of computers errors (32 significant figures with help of system REDUCE are kept up). Criterion of Fisher - Snedecor [1] for the (1) equations gives $F=74184.8>>F_{0.001;\ 2;2}=999.0$ ($F_{0.01;\ 2;1}=4999.5$), that is evidence of significance of regression equation (1). Moreover, coefficient of determination is $R^2_{n\alpha}=0.999986$ and variance is $S_e^2 = 0.0196$. It is clear, $N_t(\alpha) = 0$ for $\alpha_{10.} =-0.028915$ and $\alpha_{20}=0{,}00008414$. The minimum of the function of $N_t(\alpha)$ is $- N_{t.}(-0.014415) <0$, but usually only positive values of oblateness ($\alpha>0$) are dealt with.

In the frame of the restricted three body problems it is proved the great value of oblateness of the planet interfere with falling dawn of the noneclipticcal satellites on surface of the central body [2, 6]; Poincare's and Crudely's theorems impose restrictions on angular velocity of rotation and geometric oblateness of gravitating liquid in a state of relative equilibrium.

Basing on the table like table 1 [13] we had suggest in 2003 the hypothesis: with help of cosmic mission "Cassini" 116 satellites of Saturn would be discovered

since July 1, 2004 and the geometrical oblate nesses of Mercury and Venus are about of 0.00008 and oblateness of Pluto equals approximately 0.003. (Since 2003, 15 satellites of Saturn have been revealed).

Table (1) illustrates good agreement N_o and $N_t(\alpha)$ for the planets nearest to the Earth (and space of near which is better investigated). Since for Neptune N_o-$N_t(\alpha)$ = 0, we should wait the satellite system of Neptune are not so developed as the satellite systems of Jupiter, Saturn and Uranus.

For determination of space-time position of unknown satellites of Saturn in paper [14] new quadrature is obtained analytically. It shows dependence eccentricity (e} of osculating orbit against time (*P*).

$$2\int_{w_{min}}^{w_{max}} \frac{dw}{\sqrt{\vartheta}} = P \qquad (2)$$

$$\vartheta = \vartheta_1 \vartheta_2 \qquad (3)$$

$$\vartheta_1 = 4\gamma c_1 w^7 - \frac{4}{3}\gamma w^5 + w^2(-c_2 + 2 + 2c_1) - 4$$

$$\vartheta_2 = -4\gamma c_1 w^7 + \frac{4}{3}\gamma w^5 - 10c_1 w^4 + w^2(8 + 8c_1 + c_2) - 6$$

$$w = \frac{1}{\sqrt{1 - e^2}},$$

$c_1=(1-e^2)\cos^2 i$,

$$c_2 = \frac{2\gamma}{\sqrt{(1-e^2)^3}}\left(\frac{1}{3} + \cos 2i\right) + 2(e^2 - \sin^2 i) + e^2 \sin^2 i(5\cos 2\omega - 3)$$

$\gamma=\alpha/(2\beta)$,

$$\alpha = -\frac{3}{8} c_{20} \left(\frac{a_0}{a}\right)^2,$$

$$\beta = \frac{3}{16}\frac{\mu_1}{\mu}\left(\frac{a}{a_1}\right)^3,$$

$$\tau = \beta(t - t_0)\sqrt{\frac{\mu}{a^3}},$$

where α, β, γ, c_1, c_2, a, a_1, a_0 –are parameters of restricted twice averaged problem of 3 bodies, making into account oblateness of the planet and perturbation from the Sun; τ - modified time of motion of argument of pericentre (ω) in the interval $0<\omega<(\pi/2)$ [15].

For determination of space-time region in which Saturnine satellites may exist put the following conditions:

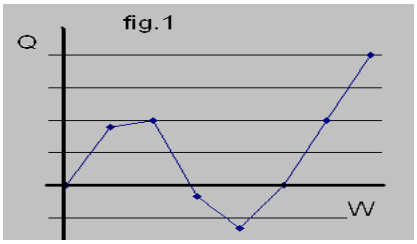
1). Pericentric distance is greater then the radius R_S of the planet;

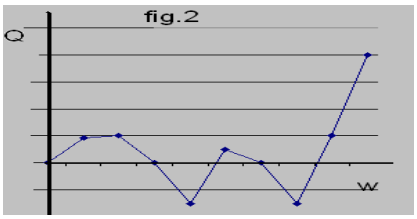
2). Apocentric distance is lesser then the sphere of action of Saturn;

3). Lifetime of satellites is greater 1 billion years.

Analyzing formula (2) the following results are drawn up. If initial meanings of $e_0=0.423009$; $i_0=53^0 6004$; $a=20R_S$, then lifetime equals 38,726 years (satellite falls down on the planet). (See fig.1)

If initial values of $e_0=0.275076$; $i_0=8^0 6534$; $a=60R_S$, then lifetime equals 18 billion years (such satellite may exist in the given model). (See fig.2)





Conclusion: The exact quantitative relationships between numbers of natural satellites of the planets of the Solar system and parameters of these planets, and theirs orbital semi major axis would make it possible to discover and investigate satellite systems of exosolar planets, because the Solar system is not unique in the Galaxy [9]. On the contrary of works M.L. Lidov and M.A. Vashkovjyak [15] our results based on the investigation of quadrature [2] derived from the system of differential equations of motion of satellite in the frame of the restricted twice-averaged 3-body problem. It is very interesting to make up a method of space-time localization of undiscovered planetary satellites based on new model problems of celestial mechanics integrated in a final form.

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